The Vibrating Inverted Pendulum

Consider a pendulum consisting of a mass attached to one end of a rigid rod with the other end of the rod attached to a pivot point which allows the mass to rotate freely in a full circle about the pivot.

If the location of the pivot is fixed then the lowest point on the circle is a stable equilibrium for the mass and the highest point is an unstable equilibrium. In this talk I will illustrate the use of “homogenization” (roughly, a refined notion of “weak” limits) to analyze the stability of the equation of motion for the pendulum (in terms of the unknown function, $\theta(t)$), assuming that the pivot point is oscillated in the vertical direction by a known function, $h(t)$. Denote the amplitude of $h$ by $A$ and the frequency by $f$. We shall see that there is a critical number $\kappa$ such that $A \cdot f > \kappa$ implies that for large enough frequencies, the high point on the circle becomes a stable equilibrium.

Watch the youtube video of an actual demonstration of this phenomena:
https://www.youtube.com/watch?v=rwGAzy0noU0