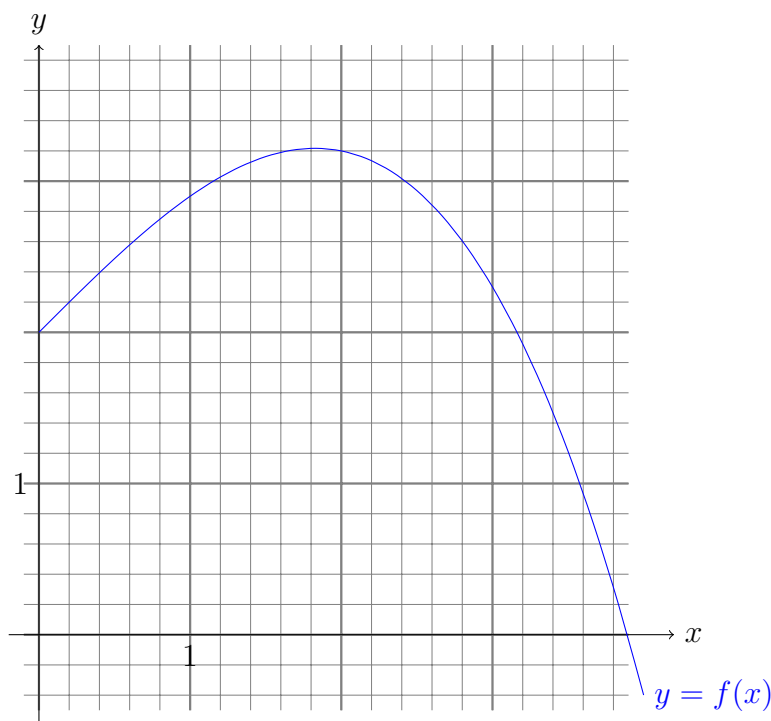


1. I am building a robot to play beanbag toss. The robot has a fixed beanbag chute, and the only way it can control where its beanbag lands is by changing the velocity: the distance the beanbag travels, as a function of initial velocity  $v$  in feet per second, is  $v^2/16$ .
  - (a) Suppose the robot needs to shoot a beanbag 27 feet. What initial velocity will accomplish this?
  - (b) Suppose the robot needs to shoot a beanbag 27 feet, and can be off by up to 3 inches. What range of initial velocities will give this outcome?
  - (c) Suppose the robot needs to shoot a beanbag 27 feet, and can be off by up to 1 inch. What range of initial velocities will give this outcome?
  - (d) Interpret parts (b) and (c) in the framework of limits: What's  $f(x)$ , what is  $a$ , what is  $L$ ? What particular values of  $\epsilon$  and  $\delta$  did you find?
  
2. A crystal growth furnace is used to grow crystals that are used in electronics. For the crystals to grow properly, the temperature must be controlled very precisely, by adjusting the input power. The power  $w$  is measured in watts, and the temperature  $T$  in degrees Celsius is given by
$$T(w) = 0.1w^2 + 2.155w + 20.$$
  - (a) How much power is needed to keep  $T$  at  $200^\circ\text{C}$ ?
  - (b) If the temperature is allowed to vary from  $200^\circ\text{C}$  by up to  $\pm 1^\circ\text{C}$ , what range of wattage is acceptable?
  - (c) Interpret part (b) in the framework of limits: What's  $f(x)$ , what is  $a$ , what is  $L$ ? What particular values of  $\epsilon$  and  $\delta$  did you find?

Here is a function  $y = f(x)$ :



3. To show that for the function defined above,

$$\lim_{x \rightarrow 1} f(x) = 2.9,$$

we need to know that for any choice of  $\epsilon$ , there exists a  $\delta$  such that if  $0 < |x - 1| < \delta$ , then  $|f(x) - 2.9| < \epsilon$ .

- Find a value of  $\delta$  that works when  $\epsilon = 0.5$ .
- Find a value of  $\delta$  that works when  $\epsilon = 0.2$ .
- Find a value of  $\delta$  that works when  $\epsilon = 0.1$ .

4. To show that for the function defined above,

$$\lim_{x \rightarrow 2} f(x) = 3.2,$$

we need to know that for any choice of  $\epsilon$ , there exists a  $\delta$  such that if \_\_\_\_\_ then \_\_\_\_\_.

- Find a value of  $\delta$  that works when  $\epsilon = 0.5$ .
- Find a value of  $\delta$  that works when  $\epsilon = 0.2$ .
- Find a value of  $\delta$  that works when  $\epsilon = 0.1$ .

5. Consider the statement that

$$\lim_{x \rightarrow 2} 3x - 5 = 1.$$

(a) Identify  $f(x)$ ,  $a$ , and  $L$ .

(b) Find a value of  $\delta$  that works when  $\epsilon = 1$ .

(c) Find a value of  $\delta$  that works when  $\epsilon = .5$ .

(d) Find a value of  $\delta$  that works when  $\epsilon = .1$ .

(e) Find a formula for  $\delta$  in terms of  $\epsilon$  that works for all potential values of  $\epsilon$ . Now prove it!