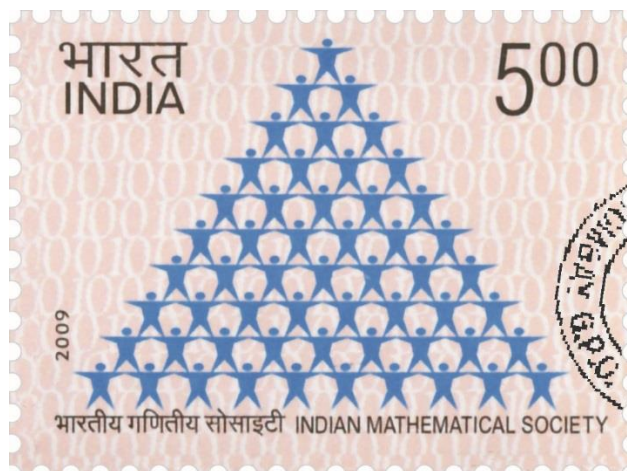# MATH 100: CLASS DISCUSSION
30 AUG 2018

Number systems & basic algebraic operations

Riddle of the day: *A frog is at the bottom of a 30 meter well. Each day he summons enough energy for one 3 meters leap up the well. Exhausted, he then hangs there for the rest of the day. At night, while he is asleep, he slips 2 meters backward. How many days does it take him to escape from the well?*

Note: Assume after the first leap that his hind legs are exactly three meters up the well. His hind legs must clear the well for him to escape.

➤ **If time permits play 2048. Download app on your phone.**

I   Section 1.1 of Larson

   (a)   What is a natural number? An integer? A rational number? An irrational number?   A real number?   Give several examples of each.

   (b)   Classify (using the terminology of (a)), each of the following numbers:

$$-1, \quad 0, \quad \pi, \quad \sqrt{2}, \quad \sqrt[3]{8}, \quad 10^9, \quad 2018, \quad 0.123123123\,\ldots, \quad \frac{17}{3}, \quad 999 + \frac{1}{2}, \quad 5 + \sqrt{3}$$

   (c) Which is larger x or y?   If x is larger than y, write $x > y$; if x is smaller than y, write $x < y$.

       (i) 14, 99

(ii) 5/7, 9/5

(iii)   90000013, -1/44

(iv) 12345/12, 7654/13

(v) $1789 - 1300 + \frac{1}{33}$, $2489 + \frac{1}{5} - 201$

(d) Find the *distance* between each of the following pairs of points on the real line:

(i) 14, 19

(ii) 19, 14

(iii) -5, 3

(iv) 0, -7/11

(v) 17.89, - 4

(vi) 8.888, 4.444

(vii) 8.888, -4.444

(vii) x, y

(viii)   x − 99, x + 67

(e) What is the meaning of *absolute value*? Evaluate each of the following:

(i) |2018|

(ii) |-2018|

(iii) -|-11|

(iv) $|5 - 11| - |-9| - 4$

(f) Express each of the following sentences as a mathematical equation.

(i) Max is at least 18 years old.

(ii) Max's cat is older than 8 years and younger than 13 years.

(iii) Albertine's laptop is less than 2.5 years old.

(g) Find two solutions to each of the following equations:

(i) $|x| = 5$

(ii) $|-y| = 5$

(iii) $|x - 9| = 4$

(iv) $|4 - 3x| = 14$

(h) Simplify each of the following:

(i) -(-9)

(ii) (-4)(-11)

(iii)   -(-1)(-2)(-3) + 8

(iv)   -(2)(-3)(4)(-5)

II Algebra exercises from Hall & Knight

1. If $a = 7$, $b = 2$, $c = 0$, $x = 5$, $y = 3$, find the value of: (A) ab^3, (B) a^4b^y,   (C) $a^5c^5y^x$,

   (D) 1^{x+3y}, (E) abcxy, (F) $2a + 5b + 9c$, (G) $3x^2 + 1$, (H) a^{b^c}, (I) $\left(a^b\right)^c$

2.   If $a = 2$, $b = 3$, $c = 1$, $p = 0$, $q = 4$, $r = 6$, find the value of: (A) $\dfrac{3a^2r}{8b}$, (B) $3a^2b^c$, (C) $\dfrac{b^r}{r^b}$,

   (D) $\dfrac{64}{q^r}$, (E) $\dfrac{5a^rb^q}{64r^a}$

3.   If $a = 2$, $b = 3$, $c = 1$, $d = 0$, find the numerical value of: (A) $3bcd + 5cda - 7dab + abc$,

   (B) $2a^2 + 3b^3 - 4c^4$,   (C) $a^2 + b^2 + c^2 + d^2$, (D) $a^4 + b^4 - c^4$

4.   If $a = 2$, $b = 1$, $c = 3$, $x = 4$, $y = 6$, $z = 0$, find the value of:

   (A) $c^2(y - x) - b^2(c - a)$,   (B) $(2a - c)(x + 2y - z)$, (C) $\dfrac{a^2}{b^2} + \dfrac{b^2}{a^2} - \dfrac{2y}{x^2}$,

   (D) $\dfrac{a^2 - b^2}{a^2b^2} - \dfrac{(a+b+z)^2}{(b+c-z)^2}$, (E) $\dfrac{(a+b+c)^2}{c(y-z)} - \dfrac{4(c-a)^3}{3(a+y)}$

5.   When x has the values 0, 3, 6, 8, 10, find the values of $x^2 - 9x + 20$.

6.   Show that, if $a = 10$ and $b = 7$, then the following two expressions are equal:

   $$4(a - b) + 3(a + b),\quad 5(a + b) + 2(a - 3b)$$
   Are these expressions equal *for all values of a and b?*

7.   When $x = 5$, show that $4x^2 + 4x - 3$ is equal to $9(x + 8)$.

vocabulary: integer, natural number, rational number, absolute value,

   algebraic expression, terms, simple expression, compound

   expression, binomial expression, substitution
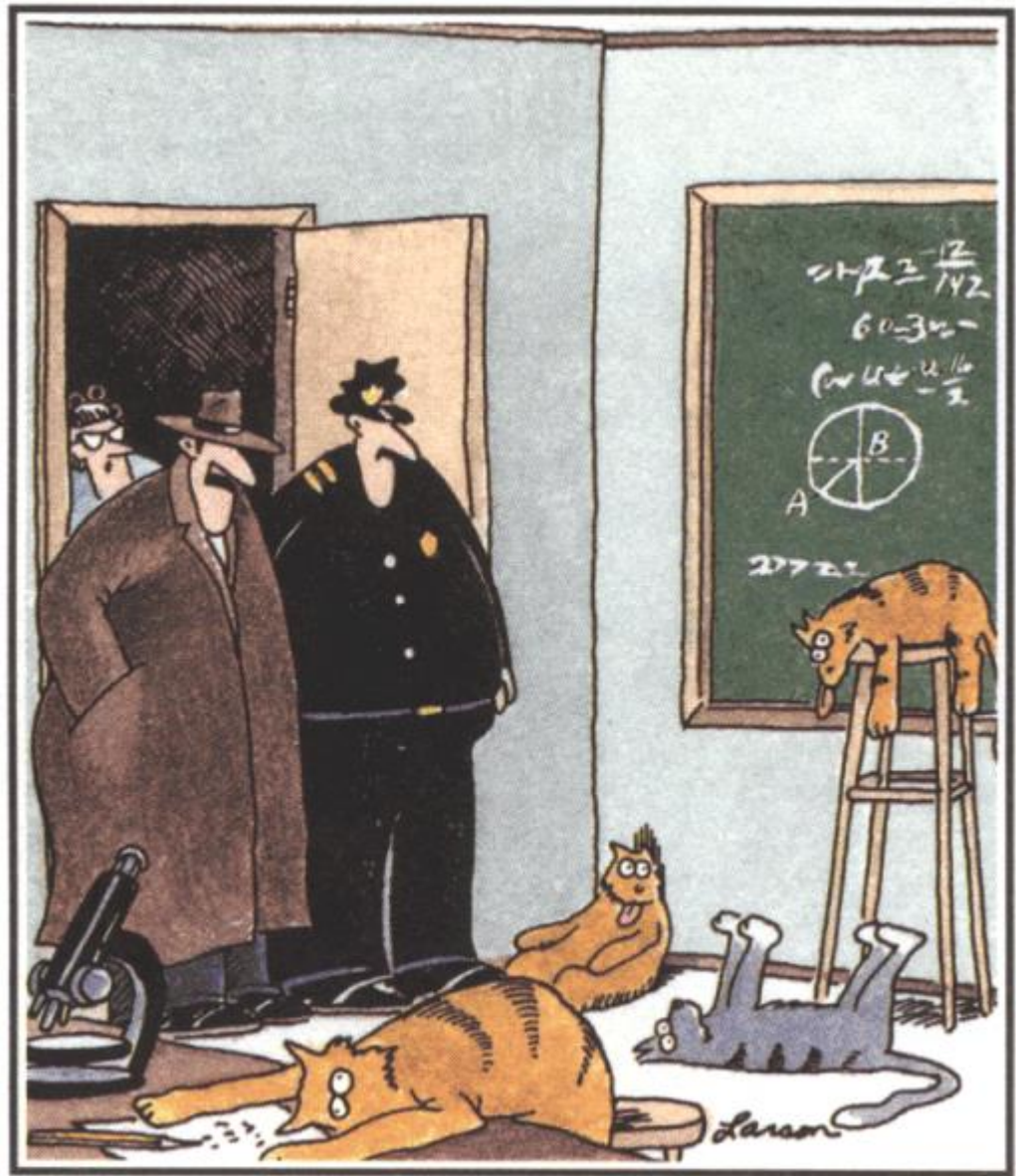
"Notice all the computations, theoretical
scribblings, and lab equipment, Norm. ...
Yes, curiosity killed these cats."