# MATH 100: CLASS DISCUSSION 

## 11 OCTOBER 2018

## LINEAR FUNCTIONS, CONTINUED



1. Identify the quadrant in which each of the following points lie:
(a) $(7,-11)$,
(b) $(-9,-9)$,
(c) $(8,3)$,
(d) $(-13,5)$,
(e) $(\pi-1, \pi-4)$
2. Find the equation of a line passing through the two points $\mathrm{P}=(1,3)$ and $\mathrm{Q}=(9,2)$. What is its slope? (a) Write this equation in slope intercept form. (b) Write this equation in point-slope form. (c) Write this equation in standard form.
3. Consider the line $y=2 x+3$. For each of the following points, determine if it lies on the line, above the line or below the line: (a) $(3,9)$, (b) $(5,4)$, (c) $(-3,0)$, (d) $(7,17),(-4,-9)$
4. Let $y=6 x+5$. Find the slope and the $y$-intercept.
5. The town of Alphaville has a population of 20,000 people. It grows by 3,000 people each year. Since the population, $P$, is growing at a constant rate of 3,000 people per year, $P$ is a linear function of time, $t$, in years.
(a) What is the rate of change of $P$ over every time interval?
(b) Create a table that gives the town's population every five years over a 25 -year period. Graph the population.
(c) Find a formula for $P$ as a function of time, $t$.
6. Albertine's new start-up company spends $\$ 40,000$ on computer equipment and, for tax purposes, chooses to depreciate it to $\$ 0$ at a constant rate over a five-year period.
(a) Create a table and a graph showing the value of the equipment over the five-year period.
(b) Find a formula for the value, $V$, of the equipment as a function of time, $t$.
7. The following table gives values of two functions, $p$ and $q$. Could either of these functions be linear? Explain.

| x | 50 | 55 | 60 | 65 | 70 |
| :--- | :--- | :--- | :--- | :--- | :--- |


| $\mathrm{p}(\mathrm{x})$ | 0.10 | 0.11 | 0.12 | 0.13 | 0.14 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{q}(\mathrm{x})$ | 0.01 | 0.03 | 0.06 | 0.14 | 0.15 |

8. Which of the following functions might be linear? Explain.
(a)

| t | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{G}(\mathrm{t})$ | 5 | 4 | 5 | 4 | 5 |

(b)

| x | 0 | 5 | 10 | 15 |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{~F}(\mathrm{x})$ | 10 | 20 | 30 | 40 |

(c)

| x | 0 | 100 | 300 | 600 |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{~g}(\mathrm{x})$ | 50 | 100 | 150 | 200 |

(d)

| x | 0 | 10 | 20 | 30 |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{~h}(\mathrm{x})$ | 20 | 40 | 50 | 55 |

(e)

| x | -3 | -1 | 0 | 3 |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{j}(\mathrm{x})$ | 5 | 1 | -1 | -7 |

(f)

| x | 9 | 8 | 7 | 6 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{p}(\mathrm{x})$ | 42 | 52 | 62 | 72 | 82 |

9. In 2010 , the population of Betaville was 17,089 and growing by 71 people each year. Find a formula, $P$, for the town's population, in terms of $t$, the number of years since 2010.
10. In 2018, the year of the zombie apocalypse, the town of Woodbury, Georgia, had 173 residents. Each year after 2018, the population fell by 13 people. Find a formula for the population of Woodbury $t$ years after 2018.
11. Odette, a woodworker, sells rocking horses. Her start-up costs, including tools, plans, and advertising, total $\$ 5,000$. Labor and materials for each horse cost $\$ 350$.
(a) Calculate Odette's total cost, $C$, to make $1,2,5,10$, and 20 rocking horses. Graph $C$ against $n$, the number of rocking horses that she carves.
(b) Find a formula for $C$ as a function of $n$.
(c) What is the rate of change of the function $C$ ? Interpret the meaning of this.
12. For each of the following linear functions, rewrite the equation in standard form and in y-intercept form.
(a) $y+3 x-3=0$
(b) $y+2(x-1)=4-11$
(c) $3 x+6=y$
(d) $1-(x-y)=4+3(1-(x-5))$
13. Find the $x$ and $y$ intercepts of each of the following straight lines:
(a) $x+5 y=19$
(b) $y-x=8$
(c) $y=7 x+9$
(d) $y=3(x-1)+7$

In Exercises 1-6, match the equation with its graph. [The graphs are labeled (a), (b), (c), (d), (e), and (f).]
(a)

(b)

(c)

(d)

(e)

(f)


1. $y=2$
2. $y=2+x$
3. $y=2-x$
4. $y=x^{2}$
5. $y=x^{2}-4$
6. $y=|x|$

In Exercises 7-30, sketch the graph of the equation. See Examples 1-3.
7. $y=3 x$
8. $y=-2 x$
9. $y=4-x$
10. $y=x-7$

In Exercises 31-44, find the $x$ - and $y$-intercepts (if any) of the graph of the equation. See Example 4.
31. $y=6 x-3$
33. $y=12-\frac{2}{5} x$
34. $y=\frac{3}{4} x+15$
35. $x+2 y=10$
36. $3 x-2 y=12$
37. $4 x-y+3=0$
38. $2 x+3 y-8=0$
93. Straight-Line Depreciation A manufacturing plant purchases a new molding machine for $\$ 230,000$. The depreciated value $y$ after $t$ years is given by
$y=230,000-25,000 t, \quad 0 \leq t \leq 8$.
Sketch a graph of this model.
94. Straight-Line Depreciation A manufacturing plant purchases a new computer system for $\$ 20,000$. The depreciated value $y$ after $t$ years is given by $y=20,000-3000 t, \quad 0 \leq t \leq 6$.

Sketch a graph of this model.
66. Height The velocity $v$ of an object projected vertically upward with an initial velocity of 64 feet per sccond is given by $v=64-32 t$, where $t$ is time in seconds. When does the object reach its maximum height?
67. Geometry The length of a rectangle is $t$ times its width (see figure). So, the perimeter $P$ is given by $P=2 w+2(n w)$, where $w$ is the width of the rectangle. The perimeter of the rectangle is 1000 meters.

(a) Complete the table of widths, lengths, and areas of the rectangle for the specified values of $t$.

| $t$ | 1 | 1.5 | 2 |
| :--- | :--- | :--- | :--- |
| Width |  |  |  |
| Length |  |  |  |
| Area |  |  |  |


| $t$ | 3 | 4 | 5 |
| :--- | :--- | :--- | :--- |
| Width |  |  |  |
| Length |  |  |  |
| Area |  |  |  |

(b) Use the table to write a short paragraph describing the relationship among the width, length, and area of a rectangle that has a fixed perimeter.
68. Geometry Repeat parts (a) and (b) of Exercise 67 for a rectangle with a fixed perimeter of 60 inches and a length that is $t$ inches greater than the width (see figure). Use the values $t=0,1,2,3,5$, and 10 to create a table given that $P=2 w+2(t+w)$.

69. Using a Model The average annual expenditures per student $y$ (in dollars) for primary and sccondary public schools in the United States from 1998 to 2005 can be approximated by the model $y=355.3 t+3725$, $8 \leq t \leq 15$, where $t$ represents the year, with $t=8$ corresponding to 1998 (see figure). According to this model, during which year did the expenditures reach $\$ 7633.30$ ? Explain how to answer the question graphically, numerically, and algebraically. (Source: National Education Association)

70. Using a Model The annual sales $y$ (in billions of dollars) of food and beverages at full-service restaurants in the United States from 1998 to 2005 can be approximated by the model $y=6.37 t+67.5$, $8 \leq t \leq 15$, where $t$ represents the year, with $t=8$ corresponding to 1998 (see figure). According to this model, in what year were the annual sales about $\$ 125$ billion? Explain how to answer the question graphically, numerically, and algebraically. (Source: National Restaurant Association)

93. Numerical Interpretation For a handyman to install $x$ windows in your home, the cost $y$ is given by $y=150 x+425$. Use $x$-values of $1,2,3,4$, and 5 to construct a table of values for $y$. Then use the table to help describe the relationship between the number of windows $x$ and the cost of installation $y$.
94. Numerical Interpretation When an employee works $x$ hours of overtime in a week, the employee's weekly pay $y$ is given by $y=18 x+480$. Use $x$-values of $0,1,2,3$, and 4 to construct a table of values for $y$. Then use the table to help describe the relationship between the number of overtime hours $x$ and the weekly pay $y$.
95. Football Pass A football quarterback throws a pass from the 10 -yard line, 10 yards from the sideline. The pass is caught by a wide receiver on the 40 -yard line, 35 yards from the same sideline, as shown in the figure. How long is the pass?

96. Soccer Pass A soccer player passes the ball from a point that is 18 yards from the endline and 12 yards from the sideline. The pass is received by a teammate who is 42 yards from the same endline and 50 yards from the same sideline, as shown in the figure. How long is the pass?


Figure for 96
97. Fuel Efficiency The scatter plot shows the speed of a car in kilometers per hour and the amount of fuel, in liters used per 100 kilometers traveled, that the car needs to maintain that speed. From the graph, how would you describe the relationship between the speed of the car and the fuel used? What is the approximate fuel efficiency if the car is traveling at 120 kilometers per hour?


Everything you've learned in school as 'obvious' becomes less and less obvious as you begin to study the universe. For example, there are no solids in the universe. There's not even a suggestion of a solid. There are no absolute continuums. There are no surfaces. There are no straight lines.

- Buckminster Fuller (1895-1983)


