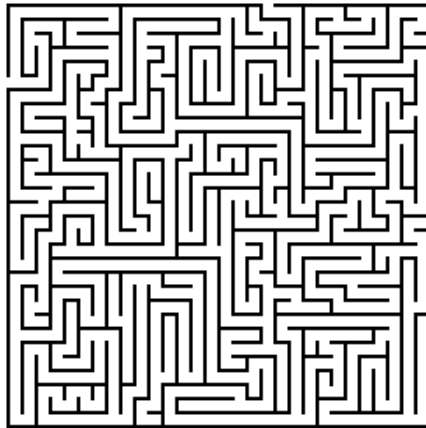


MATH 100: CLASS DISCUSSION

18 OCTOBER 2018

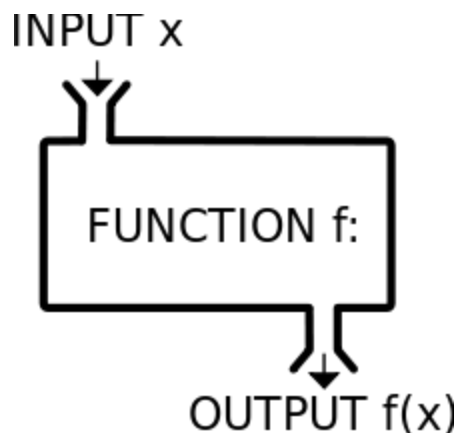


REVIEW

1. What is meant by “*slope of a straight line*”?
2. Find the slope of a line that passes through the points (1, -4) and (8, -3). Plot.
3. *Sketch* the graph of a linear function that
 - (a) passes through the points (4, 0) and (0, 8). (What is the slope of this line?)
 - (b) passes through the points (1, 1) and (-3, -4). (What is the slope of this line?)
 - (c) passes through the point (1, 2) and has slope 3
 - (d) passes through the point (0, 3) and has slope -4.
 - (e) passes through the point (1, 2) and has slope $7/2$.
4. Find the x and y intercepts of the line $3x + 8y = 3$.
5. Find the x and y intercepts of the line $y = 4x - 13$
6. Which of the following triples of points are *collinear*? Plot each triple.
 - (a) $P = (2, 6)$, $Q = (5, 2)$, $R = (8, -2)$
 - (b) $P = (2, 3)$, $Q = (2, 6)$, $R = (6, 3)$
 - (c) $P = (8, 3)$, $Q = (5, 2)$, $R = (2, 1)$
 - (d) $P = (2, 4)$, $Q = (1, 1)$, $R = (0, -2)$
7. Find an equation of a line that is *parallel* to $2x + 4y = 1$ and passes through (-3, -5).
8. Are the following lines *perpendicular*? Why?
 $3x - 5y = 1$ and $10x + 6y = 7$
9. Find an equation of a line that passes through $(1, \frac{1}{2})$ and is perpendicular to the line $y - 3x = 4$.
10. Write each of the following in *slope-intercept* form:
 - (a) $1 - x - 5y = 8$
 - (b) $3(2 - y) = 1 + 4(x - 2)$
 - (c) $2(1 - 3(1 - x - y)) = x + y$
 - (d) $2y + x = -(4 - y - x) + 9$
11. Use the point-slope form to find an equation of a line with the given properties:
 - (a) slope = 7, through $P = (3, 1)$
 - (b) slope = $-4/5$, through $Q = (-1, -2)$
 - (c) through $P = (1, 3)$ and $Q = (5, 9)$
12. Find t such that the point $P = (t, 5)$ lies on the line of slope $m = -3$ that passes through the point (7, 11).
13. For each of the following linear functions, rewrite the equation in standard form and in y -intercept form.
 - (a) $y + 3x - 3 = 0$
 - (b) $y + 2(x - 1) = 4 - 11$

- (c) $3x + 6 = y$
 (d) $1 - (x - y) = 4 + 3(1 - (x - 5))$
14. Find the x and y intercepts of each of the following straight lines:
 (a) $x + 5y = 19$
 (b) $y - x = 8$
 (c) $y = 7x + 9$
 (d) $y = 3(x - 1) + 7$
15. The town of Alphaville has a population of 20,000 people. It grows by 3,000 people each year. Since the population, P , is growing at a constant rate of 3,000 people per year, P is a linear function of time, t , in years.
 (a) What is the rate of change of P over every time interval?
 (b) Create a table that gives the town's population every five years over a 25-year period. Graph the population.
 (c) Find a formula for P as a function of time, t .
16. Albertine's new start-up company spends \$40,000 on computer equipment and, for tax purposes, chooses to depreciate it to \$0 at a constant rate over a five-year period.
 (a) Create a table and a graph showing the value of the equipment over the five-year period.
 (b) Find a formula for the value, V , of the equipment as a function of time, t .
17. Find the *distance* between the following pairs of points. Also plot the points.
 (a) $P = (2018, 77)$, $Q = (2018, 97)$
 (b) $P = (5, 44)$, $Q = (9, 44)$
 (c) $P = (1, 1)$, $Q = (4, 5)$
 (d) $P = (-3, 4)$, $Q = (4, 5)$
 (e) $P = (99, -104)$, $Q = (100, -101)$
18. Find the *perimeter* of the triangle with vertices $A = (-1, -1)$, $B = (0, 5)$, $C = (4, 4)$.
19. Odette, a woodworker, sells rocking horses. Her start-up costs, including tools, plans, and advertising, total \$5,000. Labor and materials for each horse cost \$350.
 (a) Calculate Odette's total cost, C , to make 1, 2, 5, 10, and 20 rocking horses. Graph C against n , the number of rocking horses that she carves.
 (b) Find a formula for C as a function of n .
 (c) What is the rate of change of the function C ? Interpret the meaning of this.

INTRODUCTION TO FUNCTIONS



1. Write each of the following relationships using function notation:
 (a) Weight, w , is a function of caloric intake, c .
 (b) The number of molecules, m , in a gas, is a function of the volume of the gas, v .
 (c) The final exam grade, G , of a student is a function of the number of hours, t , of sleep s/he has the night before the final.

2. Let $f(t)$ be the number of people in the U.S., in millions, who own cell phones t years after 1990. Explain the meaning of each of the following statements:

- (a) $f(9) = 100.3$; (b) $f(a) = 20$; (c) $f(25) = b$; (d) $n = f(t)$

3. Suppose that you ride on a Ferris wheel beginning at time zero, and remain on the wheel for three complete revolutions before exiting. Draw a graph of your *height above the ground* as a function of time. Be certain to label the axes, and to define the units.

4. Sketch a curve of the height of a person from time of birth until time of death. What is the independent variable? the dependent variable?

5. The population, P , of the land of Oz in millions, is a function of t , the number of years since 1980; so $P = f(t)$. Explain the meaning of the statement $f(45) = 12$

6. Let $g(x) = 3x + 5$ be a function. Compute and simplify each of the following:

- (a) $g(3)$
(b) $g(-9)$
(c) $g(3x)$
(d) $g(x^3)$
(e) $g(5 + z)$
(f) $\{g(x + h) - g(x)\} / h$

7. Let $f(x) = x^2 - 4x + 5$ be a function.

- (a) Find $f(-1)$, $f(0)$, $f(3)$, and $f(0.1)$.
(b) Find $f(2b)$
(c) Find $f(c - 1)$
(d) Find $f(x^2)$
(e) Find $f(1+h) - f(1)$
(f) Simplify $\{f(1 + h) - f(1 - h)\}/h$

Solving Problems

83. **▲ Geometry** Write the perimeter P of a square as a function of the length x of one of its sides.

84. **▲ Geometry** Write the surface area S of a cube as a function of the length x of one of its edges.

85. **▲ Geometry** Write the volume V of a cube as a function of the length x of one of its edges.

86. **▲ Geometry** Write the length L of the diagonal of a square as a function of the length x of one of its sides.

87. **Cost** The cost of producing a video game is \$1.95 per unit with fixed costs of \$8000. Write the total cost C as a function of x , the number of units produced.

88. **Cost** The cost of producing a software program is \$3.25 per unit with fixed costs of \$495. Write the total cost C as a function of x , the number of units produced.

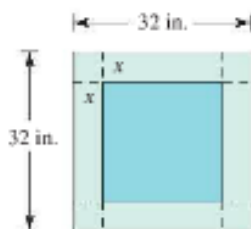
89. **Distance** An airplane flies at a speed of 120 miles per hour. Write the distance d traveled by the airplane as a function of time t in hours.

90. **Distance** A car travels for 4 hours on a highway at a steady speed. Write the distance d traveled by the car as a function of its speed s in miles per hour.

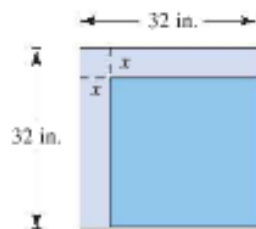
91. **Distance** A train travels at a speed of 65 miles per hour. Write the distance d traveled by the train as a function of time t in hours. Then find d when the value of t is 4.

92. **Distance** A migrating bird flies at a steady speed for 8 hours. Write the distance d traveled by the bird as a function of its speed s in miles per hour. Then find d when the value of s is 35.

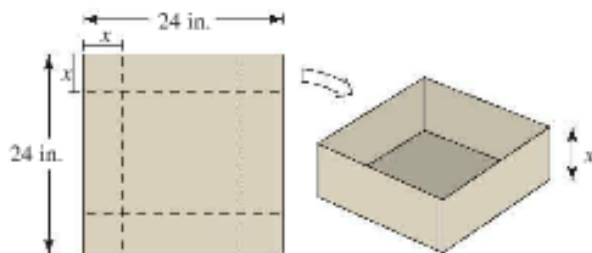
93. **▲ Geometry** Strips of width x are cut from the four sides of a square that is 32 inches on a side (see figure). Write the area A of the remaining square as a function of x .



94. **▲ Geometry** Strips of width x are cut from two adjacent sides of a square that is 32 inches on a side (see figure). Write the area A of the remaining square as a function of x .



95. **▲ Geometry** An open box is to be made from a square piece of material 24 inches on a side by cutting equal squares from the corners and turning up the sides (see figure). Write the volume V of the box as a function of x .



96. **▲ Geometry** You use the method of Exercise 95 to make a box from a rectangular piece of material that is 24 inches long and 20 inches wide. Write the volume of the box as a function of x .

97. **Profit** The marketing department of a business has determined that the profit from selling x units of a product is approximated by the model

$$P(x) = 50\sqrt{x} - 0.5x - 500.$$

Find (a) $P(1600)$ and (b) $P(2500)$.

98. **Safe Load** A solid rectangular beam has a height of 6 inches and a width of 4 inches. The safe load S of the beam with the load at the center is a function of its length L and is approximated by the model

$$S(L) = \frac{128,160}{L}$$

where S is measured in pounds and L is measured in feet. Find (a) $S(12)$ and (b) $S(16)$.

Wages In Exercises 99 and 100, use the following information. A wage earner is paid \$12.00 per hour for regular time and time-and-a-half for overtime. The weekly wage function is

$$W(h) = \begin{cases} 12h, & 0 \leq h \leq 40 \\ 18(h - 40) + 480, & h > 40, \end{cases}$$

where h represents the number of hours worked in a week.

99. (a) Evaluate $W(30)$, $W(40)$, $W(45)$, and $W(50)$.
- (b) Could you use values of h for which $h < 0$ in this model? Why or why not?
100. (a) Evaluate $W(20)$, $W(25)$, $W(35)$, and $W(55)$.
- (b) Describe the domain implied by the situation.

Data Analysis In Exercises 101 and 102, use the graph, which shows the numbers of students (in millions) enrolled in public and private post-secondary degree-granting institutions in the United States. (Source: U.S. National Center for Education Statistics)

101. Is the enrollment in public post-secondary schools a function of the year? Explain. Let $f(x)$ represent the number of students in these schools in year x . Approximate $f(2004)$.
102. Is the enrollment in private post-secondary schools a function of the year? Explain. Let $g(x)$ represent the number of students in these schools in year x . Approximate $g(2003)$.

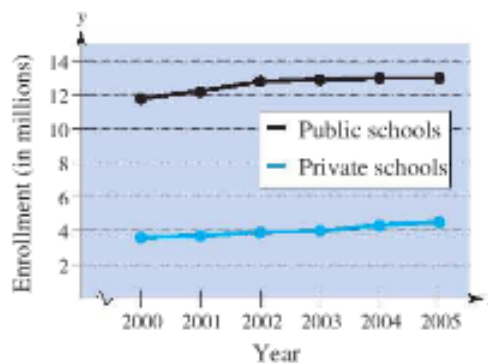


Figure for 101 and 102

Vocabulary: function, independent variable, dependent variable, domain, range, vertical line test

Life and death are one thread, the same line viewed from different sides.

- Lao Tzu