

MATH 100: CLASS DISCUSSION

2 OCTOBER 2018

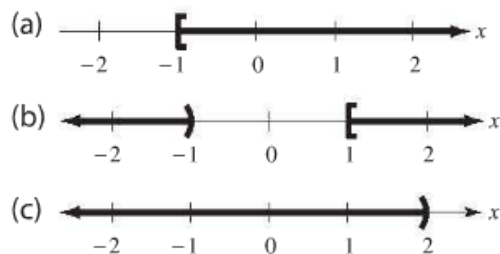
Inequalities, absolute values

Developing Skills

In Exercises 1–4, determine whether each value of x satisfies the inequality.

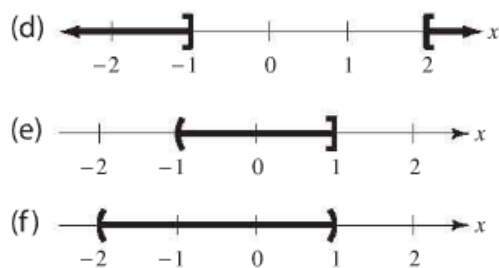
- | <i>Inequality</i> | <i>Values</i> | |
|--------------------------------|-----------------------|-----------------------|
| 1. $7x - 10 > 0$ | (a) $x = 3$ | (b) $x = -2$ |
| | (c) $x = \frac{5}{2}$ | (d) $x = \frac{1}{2}$ |
| 2. $3x + 2 < \frac{7x}{5}$ | (a) $x = 0$ | (b) $x = 4$ |
| | (c) $x = -4$ | (d) $x = -1$ |
| 3. $0 < \frac{x+4}{5} < 2$ | (a) $x = 10$ | (b) $x = -4$ |
| | (c) $x = 0$ | (d) $x = 6$ |
| 4. $-3 < \frac{2-x}{2} \leq 3$ | (a) $x = 0$ | (b) $x = 7$ |
| | (c) $x = 9$ | (d) $x = -1$ |

In Exercises 5–10, match the inequality with its graph. [The graphs are labeled (a), (b), (c), (d), (e), and (f).]



In Exercises 27–34, determine whether the inequalities are equivalent.

27. $3x - 2 < 12, 3x < 10$
28. $6x + 7 \geq 11, 6x \geq 18$
29. $-5(x + 12) > 25, x + 12 > -5$
30. $-4(5 - x) < 32, 5 - x < -8$
31. $7x - 6 \leq 3x + 12, 4x \leq 18$
32. $11 - 3x \geq 7x + 1, 10 \geq 10x$
33. $3x > 5x, 3 > 5$
34. $4x > -8x, -4 < 8$



5. $x \geq -1$
6. $-1 < x \leq 1$
7. $x \leq -1$ or $x \geq 2$
8. $x < -1$ or $x \geq 1$
9. $-2 < x < 1$
10. $x < 2$

In Exercises 11–24, sketch the graph of the inequality. See Example 1.

11. $x \leq 4$
12. $x > -6$
13. $x > 3.5$
14. $x \leq -2.5$
15. $-5 < x \leq 3$
16. $-1 < x \leq 5$
17. $4 > x \geq 1$
18. $9 \geq x \geq 3$
19. $\frac{3}{2} \geq x > 0$
20. $-\frac{15}{4} < x < -\frac{5}{2}$
21. $x < -5$ or $x \geq -1$
22. $x \leq -4$ or $x > 0$
23. $x \leq 3$ or $x > 7$
24. $x \leq -1$ or $x \geq 1$

25. Write an inequality equivalent to $5 - \frac{1}{3}x > 8$ by multiplying each side by -3 .
26. Write an inequality equivalent to $5 - \frac{1}{3}x > 8$ by adding $\frac{1}{3}x$ to each side.

67. $-3 < \frac{2x - 3}{2} < 3$

68. $0 \leq \frac{x - 5}{2} < 4$

69. $1 > \frac{x - 4}{-3} > -2$

70. $-\frac{2}{3} < \frac{x - 4}{-6} \leq \frac{1}{3}$

71. $2x - 4 \leq 4$ and $2x + 8 > 6$

72. $7 + 4x < -5 + x$ and $2x + 10 \leq -2$

73. $8 - 3x > 5$ and $x - 5 \geq 10$

74. $9 - x \leq 3 + 2x$ and $3x - 7 \leq -22$

75. $7.2 - 1.1x > 1$ or $1.2x - 4 > 2.7$

In Exercises 35–82, solve the inequality and sketch the solution on the real number line. *See Examples 2–8.*

35. $x - 4 \geq 0$

36. $x + 1 < 0$

37. $x + 7 \leq 9$

38. $z - 4 > 0$

39. $2x < 8$

40. $3x \geq 12$

41. $-9x \geq 36$

42. $-6x \leq 24$

43. $-\frac{3}{4}x < -6$

44. $-\frac{1}{5}x > -2$

45. $5 - x \leq -2$

46. $1 - y \geq -5$

47. $2x - 5.3 > 9.8$

48. $1.6x + 4 \leq 12.4$

49. $5 - 3x < 7$

50. $12 - 5x > 5$

51. $3x - 11 > -x + 7$

52. $21x - 11 \leq 6x + 19$

53. $-3x + 7 < 8x - 13$

54. $6x - 1 > 3x - 11$

55. $\frac{x}{4} > 2 - \frac{x}{2}$

56. $\frac{x}{6} - 1 \leq \frac{x}{4}$

57. $\frac{x-4}{3} + 3 \leq \frac{x}{8}$

58. $\frac{x+3}{6} + \frac{x}{8} \geq 1$

59. $\frac{3x}{5} - 4 < \frac{2x}{3} - 3$

60. $\frac{4x}{7} + 1 > \frac{x}{2} + \frac{5}{7}$

61. $0 < 2x - 5 < 9$

62. $-6 \leq 3x - 9 < 0$

63. $8 < 6 - 2x \leq 12$

64. $-10 \leq 4 - 7x < 10$

65. $-1 < -0.2x < 1$

66. $-2 < -0.5s \leq 0$

76. $0.4x - 3 \leq 8.1$ or $4.2 - 1.6x \leq 3$

77. $7x + 11 < 3 + 4x$ or $\frac{5}{2}x - 1 \geq 9 - \frac{3}{2}x$

78. $3x + 10 \leq -x - 6$ or $\frac{1}{2}x + 5 < \frac{5}{2}x - 4$

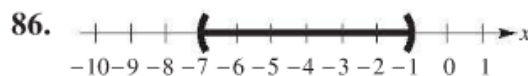
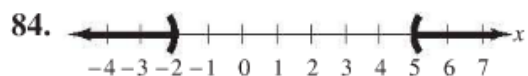
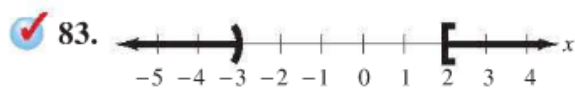
79. $-3(y + 10) \geq 4(y + 10)$

80. $2(4 - z) \geq 8(1 + z)$

81. $-4 \leq 2 - 3(x + 2) < 11$

82. $16 < 4(y + 2) - 5(2 - y) \leq 24$

In Exercises 83–88, write the solution set as a compound inequality. Then write the solution using set notation and the union or intersection symbol. *See Example 9.*



Solving Problems

- 105. Budget** A student group has \$4500 budgeted for a field trip. The cost of transportation for the trip is \$1900. To stay within the budget, all other costs C must be no more than what amount?
- 106. Budget** You have budgeted \$1800 per month for your total expenses. The cost of rent per month is \$600 and the cost of food is \$350. To stay within your budget, all other costs C must be no more than what amount?
- 107. Meteorology** Miami's average temperature is greater than the average temperature in Washington, DC, and the average temperature in Washington, DC is greater than the average temperature in New York City. How does the average temperature in Miami compare with the average temperature in New York City?
- 108. Elevation** The elevation (above sea level) of San Francisco is less than the elevation of Dallas, and the elevation of Dallas is less than the elevation of Denver. How does the elevation of San Francisco compare with the elevation of Denver?

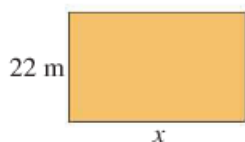


Figure for 113

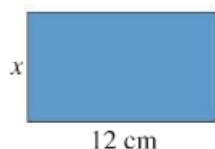


Figure for 114

- 114. Geometry** The length of a rectangle is 12 centimeters. The perimeter of the rectangle must be at least 30 centimeters and not more than 42 centimeters. Find the interval for the width x .
- 115. Number Problem** Four times a number n must be at least 12 and no more than 30. What interval represents the possible values of n ?
- 116. Number Problem** What interval represents the values of n for which $\frac{1}{3}n$ is no more than 9?
- 117. Hourly Wage** Your company requires you to select one of two payment plans. One plan pays a straight \$12.50 per hour. The second plan pays \$8.00 per hour plus \$0.75 per unit produced per hour. Write an inequality for the number of units that must be produced per hour so that the second option yields the greater hourly wage. Solve the inequality.

- 109. Operating Costs** A utility company has a fleet of vans. The annual operating cost per van is $C = 0.35m + 2900$, where m is the number of miles traveled by a van in a year. What is the maximum number of miles that will yield an annual operating cost that is no more than \$12,000?
- 110. Operating Costs** A fuel company has a fleet of trucks. The annual operating cost per truck is $C = 0.58m + 7800$, where m is the number of miles traveled by a truck in a year. What is the maximum number of miles that will yield an annual operating cost that is less than \$25,000?

Cost, Revenue, and Profit In Exercises 111 and 112, the revenue R from selling x units and the cost C of producing x units of a product are given. In order to obtain a profit, the revenue must be greater than the cost. For what values of x will this product produce a profit?

111. $R = 89.95x$ **112.** $R = 105.45x$
 $C = 61x + 875$ $C = 78x + 25,850$

- 113. Geometry** The width of a rectangle is 22 meters. The perimeter of the rectangle must be at least 90 meters and not more than 120 meters. Find the interval for the length x (see figure on page 106).

- 118. Monthly Wage** Your company requires you to select one of two payment plans. One plan pays a straight \$3000 per month. The second plan pays \$1000 per month plus a commission of 4% of your gross sales. Write an inequality for the gross sales per month for which the second option yields the greater monthly wage. Solve the inequality.

Energy In Exercises 119 and 120, use the equation $y = 21.8t - 160$, for $9 \leq t \leq 15$, which models the annual consumption of energy produced by wind (in trillions of British thermal units) in the United States from 1999 to 2005. In this model, t represents the year, with $t = 9$ corresponding to 1999. (Source: U.S. Energy Information Administration)

- 119.** During which years was the consumption of energy produced by wind less than 100 trillion Btu?
- 120.** During which years was the consumption of energy produced by wind greater than 130 trillion Btu?

Developing Skills

In Exercises 1–4, determine whether the value is a solution of the equation.

<i>Equation</i>	<i>Value</i>
1. $ 4x + 5 = 10$	$x = -3$
2. $ 2x - 16 = 10$	$x = 3$
3. $ 6 - 2w = 2$	$w = 4$
4. $ \frac{1}{2}t + 4 = 8$	$t = 6$

In Exercises 5–8, transform the absolute value equation into two linear equations.

5. $|x - 10| = 17$
6. $|7 - 2t| = 5$
7. $|4x + 1| = \frac{1}{2}$
8. $|22k + 6| = 9$

In Exercises 9–12, write the absolute value equation in standard form.

- | | |
|---------------------|-------------------------------|
| 9. $ 3x + 7 = 8$ | 10. $ 5x - 6 = -3$ |
| 11. $3 2x - 1 = 5$ | 12. $\frac{1}{4} 3x + 1 = 4$ |

In Exercises 13–40, solve the equation. (Some equations have no solution.) *See Examples 1–5.*

- | | |
|-------------------|--------------------------|
| 13. $ x = 4$ | 14. $ x = 3$ |
| 15. $ t = -45$ | 16. $ s = 16$ |
| 17. $ h = 0$ | 18. $ x = -82$ |
| 19. $ 5x = 15$ | 20. $ \frac{1}{3}x = 2$ |
| 21. $ x + 1 = 5$ | 22. $ x + 5 = 7$ |

25. $|4 - 3x| = 0$

26. $|3x - 2| = -5$

27. $|5x - 3| + 8 = 22$

28. $|5 - 2x| + 10 = 6$

29. $|\frac{x-2}{3}| + 6 = 6$

30. $|\frac{x-2}{5}| + 4 = 4$

31. $-2|7 - 4x| = -16$

32. $4|5x + 1| = 24$

33. $3|2x - 5| + 4 = 7$

34. $2|4 - 3x| - 6 = -2$

35. $|x + 8| = |2x + 1|$

36. $|10 - 3x| = |x + 7|$

37. $|3x + 1| = |3x - 3|$

38. $|2x + 7| = |2x + 9|$

39. $|4x - 10| = 2|2x + 3|$

40. $3|2 - 3x| = |9x + 21|$

Think About It In Exercises 41 and 42, write an absolute value equation that represents the verbal statement.

41. The distance between x and 4 is 9.

42. The distance between -3 and t is 5.

In Exercises 43–46, determine whether the x -value is a solution of the inequality.

<i>Inequality</i>	<i>Value</i>
43. $ x < 3$	$x = 2$
44. $ x \leq 5$	$x = -7$