MATH 100 PRACTICE PROBLEMS FOR TEST III

"It's just a simple Rorschach ink-blot test, Mr. Bromwell, so just calm down and tell me what each one suggests to you."

1. Find the domain of each of the following functions:

$(a)\ \ g(x) = \dfrac{7-5x}{(x-)(x-8)(x-1)}$

$(b)\ \ g(x) = \sqrt{x} + 3\sqrt{17-x}$

2 . Express in scientific notation: $\dfrac{7.4\times10^{38}}{9.3\times10^{57}}$

3. Let $f(x) = 2x^2 - 3x$. Find and simplify:

(a) f(-1)

(b) f(a – b)

(c) f(x + h)

(d) f(x – 2h)

(e) f(a + b + c)

4. Find the ***domain*** of each of the following functions:

(a) $y = 7x + 19$

(b) $y = 1 + \frac{1}{x}$

(c) $y = 12(x + 3)^5$

(d) $y = \frac{x}{x-3}$

(e) $y = 5|2x - 1|$

(f) $y = \sqrt{1 - 4x}$

(g) $y = \sqrt[3]{1 - 4x}$

(h) $y = \frac{(x+1)(x+2)}{(x+3)(x+4)(x+5)}$

(i) $y = \sqrt{x^2 + 1}$

5. Use 2 unknowns: The sum of the ages of Pozzo and Vladimir is 61 years. Twenty years from now (provided they survive) Vladimir will be 26 years less than twice Pozzo's age. Find their current ages.

6. Solve each of the following:

(a) $|3x - 4| = |5 - x|$

(b) $|4x + 13| = -1789$

7. Express each of the following sums or differences in scientific notation.

(a) $4.9 \times 10^2 + 7.9 \times 10^3$

(b) $4.9 \times 10^{-6} - 7.9 \times 10^{-5}$

8. Factor fully: (a) $x^8 - 1$ (b) $4(x - y)^2 - (x - y)$ (c) $500x^2y - 20y^3$

(d) $3x^2 - 5x - 2$ (e) $4x^2 + 12x + 9$ (f) $9x^2 - 49$

(g) $a^3 - ax$ (h) $2019a^2 - 333a$ (i) $5a^2bx^3 - 15abx^2 - 20b^3x^2$

(j) $500x^2y - 20y^3$

9. Factor: (a) $6x^2 - 11x - 10$, (b) $3x^2 - 17x^2 - 28$, (c) $4x^2 + 47x + 33$

(d) $x^2 - 11x + 30$ (e) $x^2 - x - 240$ (f) $x^2 - x - 240$

(g) $x^2 - 5xy - 24y^2$ (h) $x^2 - 20xy - 96y^2$

10. Express each product or quotient in scientific notation.

1. $(2.85\times10^{7})(3.16\times10^{-3})$

2. $\dfrac{9.12\times10^{-5}}{3.8\times10^{6}}$

3. $\dfrac{(6.2\times10^{3})(3.8\times10^{5})}{1.9\times10^{4}}$

4. $\dfrac{(1.5\times10^{4})(6.25\times10^{-3})}{1.25\times10^{-4}}$

5. $\dfrac{(5.06\times10^{6})(8.24\times10^{-3})}{2.2\times10^{5}}$

11. Which of the following numbers are divisible by 3? (a) 33333 (b) 123450 (c) 718191001

12. Factor and simplify: (A) $(3a + 1)^2 - (2a - 1)^2$ (B) $(a + b)^4 - 1$

(C) $1 - (x - y)^2$

13. Solve using scientific notation.

(a) Light travels art approximately 3.0×10^8 m/sec. How far does light travel in one week?

(b) Assume that there are 20,000 runners in the New York City Marathon. Each runner runs a distance of 26 miles. If you add together the total number of miles for all runners, how many times around the globe would the maratón runners have gone? Hint: Assume that the circumference of the earth is 2.5×10^4 miles.

(c) Mercury's average distance from the sun is 57,910,000 km. The Earth is approximately 93,000,000 miles from the sun. What would be the distance of a trip from Mercury to Earth via the Sun? (Note: 1 km = 0.62 miles)

14. Find the gcd and lcm of: (a) 77 and 33 (b) 880, 50, 320]

15. Find gcd and lcm of: (a) $x(x+1),\ x(x+2)^3,\ 4x^2(x+2)^2$

(b) $5(x+7)(x+9),\quad (x+9)(x+10),\quad 4(x+7)(x+9)^{33}$

16. Let $p(x) = 3x^3 - 3x + 1$ and let $q(x) = x^2 - x - 1$.

(a) Compute $q - 2p$ and express it as a polynomial in standard form (that is, decreasing exponents).

(b) Compute pq and express it as a polynomial in standard form (that is, decreasing exponents).

17. Without using your calculator, compute $80001^2 - 79999^2$ *showing all of your steps*.

18. Find all the roots of the polynomial

$$f(x) = (x^2 + 1)(x^2 - 4)(x^2 + 9)(x^2 - 16)(4x^2 - 16x + 15)$$

19. Find the *domain* of the function $f(x) = \dfrac{17(x-4)(x+9)(x^2+1)(3x-1)}{(x-1)^3(x+1)(x^4+99)(2x+5)}$

20. Perform the indicated operations of multiplication or division.

(a) $\dfrac{x-5}{7} \cdot \dfrac{x^2-2x}{x^2-7x+10}$

(b) $\dfrac{x^2+3x-10}{5x} \cdot \dfrac{x^2-3x}{x^2-5x+6}$

(c) $\dfrac{a-b}{3(a+b)} \div \dfrac{a^2-b^2}{a^2+2a+1}$

(d) $\dfrac{2x^2+8x+8}{x^2+4x+4} \cdot \dfrac{2x^2+7x+6}{4x^2+14x+12}$

(e) $\dfrac{2x^4+4x^2}{6x^2+14x+4} \div \dfrac{x^2+2}{3x^2+x}$

(f) $\dfrac{2x^2+3x}{4x^2} \cdot \dfrac{4x^2-6x}{12x+18}$

(g) $\dfrac{2x^2+5x+2}{x^2-4} \cdot \dfrac{x^2+4x}{2x^2+9x+4}$

(h) $\dfrac{x^2-14x-15}{x^2-4x-45} \div \dfrac{x^2-12x-45}{x^2-6x-27}$

(i) $\dfrac{x^2+3x+2}{x^2+9x+20} \cdot \dfrac{x^2+7x+12}{x^2+5x+6}$

(j) $\dfrac{5x}{6(x^2-1)} - \dfrac{1}{2(x-1)} + \dfrac{1}{3(x+1)}$

(k) $\dfrac{10}{x^2-1} + \dfrac{2}{x+3} - \dfrac{1}{x-3}$

(l) $\dfrac{1}{x^2-7x+12} - \dfrac{1}{x^2-5x+6}$

(m) $4-\dfrac{x-1}{x^2+3x-10}$

(n) $\dfrac{3x+2}{4x+1}-\dfrac{3x+6}{4x^2+9x+2}$

(o) $\dfrac{-x^2+5x}{(x-5)^2}+\dfrac{x+8}{x-5}$

21. Evaluate each of the following.

(a) $64^{5/3}$

(b) $(625/121)^{-1/2}$

(c) $(25/16)^{-3/2}$

22. Express with *positive* indices.

(a) $c^{-1}\, b^{-9/4}\, c^{5/6}$

(b) $a^{-5}\, b^{-9}/c^{-3}$

(c) $4x^{-3}y^{5} / (24a^{7}y^{-8})$

23. Solve for *x*:

$$\frac{x^2}{x-7}=\frac{121}{x-7}$$

24. Let $p(x) = 5x^4 + x^2 + x + 3$ and let $q(x) = x^4 - 2x^3 - x^2 - 5$. Compute pq and express it as a polynomial in standard form (that is, decreasing exponents).

25. Solve for *x*:

$$\frac{x-2}{3x-8}=\frac{x-4}{3x-4}$$

26. Solve for *x*: $\dfrac{4(x+2)}{5}=7+\dfrac{5x}{13}$

27. Use two unknowns: Gilberte bought 135 pieces of candy to give away on Halloween. She bought two kinds of Candy, paying 18 cents each for Tootsie Rolls and 24 cents a piece for chocolate skeletons. If she spent $26.70 for the Candy, how many pieces of each kind did she buy? *(Need not solve.)*

28. Use two unknowns: Albertine earns twice as much per hour for tutoring math as she does working at Starbuck's. If her average wage is $11.25 per hour, how much does she earn per hour at each job? *(Need not solve.)*

29. Use two unknowns: Last year, Madame Vedurin invested 50,000 euros. She invested part of her money in a real estate venture in Bretagne that paid 7.5% for the year and the rest in a café venture that returned 12% for the year. The combined income from the two investments for the year totaled 5190 euros. How much did she invest at each rate? *(Need not solve.)*

30. Let $p(x) = x^2 - x + 2$ and $q(x) = x^3 + x^2 + x + 1$. Compute and simplify:

(a) $3p$ (b) $q - 4p$ (c) p^2 (d) pq

(e) q^2 (f) $p(x + 1) - p(x)$

31. Use two unknowns: Swann, a fruit dealer, paid a total of $67 for strawberries and peaches. He sold the strawberries at a profit of 20% on the cost and the peaches at a loss of 2% on the cost. If his total profit was $8.56, how much did he pay for each kind of fruit?

32. Find the roots of the polynomials:

(a) $y = x^3 (x - 2)^5 (x + 7)^5(x^2 + 1)$, (b) $y = (x^2 - 14x - 15)^4$, (c) $y = x^8 - 4x^6$,

(d) $y = (x - 9)(x - 5) - (x - 9)(2x - 8)$

33. Use two unknowns: The length of a room exceeds its width by 7 feet; if each dimension had been increased by 1 foot, the area would have been increased by 51 square feet; find the original dimensions of the room.

34. Use two unknowns: Sir Charles exercises every day. He walks at 3 miles per hour and then jogs at 5 miles per hour. If it takes him 0.9 hours to travel a total of 3.5 miles, how long does he jog?

35. Using Gaussian elimination solve the system:

$9x - 7y = 4$

$6x + 3y = 1$

36. *Simplify each of the following*:

(a) $\dfrac{6/5}{9/25}$ (b) $\dfrac{\frac{ab}{cd}}{\frac{a+b}{cd}}$ (c) $\dfrac{\frac{abc}{bc^2d}}{\frac{ab(c+d)}{bc}}$

37. Using Gaussian elimination solve:

$9x - 7y = 4$

$6x + 3y = 1$

38.

[16 points] Marisa is planning to open a lemonade stand, and she needs to buy equipment and ingredients to make the lemonade. If she decides to make a total of 12 gallons of lemonade, the equipment and ingredients will cost her a total of 57 dollars. However, if she decides to make 20 gallons, it will cost her 85 dollars.

a. [5 points] Let $C(g)$ be the cost to Marisa, in dollars, of producing g gallons of lemonade. Assuming $C(g)$ is a linear function, find a formula for $C(g)$.

b. [3 points] Find and give a practical interpretation, in the context of this problem, of the slope of the function $C(g)$. *Include units.*

c. [2 points] Find the vertical intercept of the function $C(g)$. *Include units.*

d. [3 points] Marisa sells lemonade for 25 cents per cup (there are 16 cups in one gallon of lemonade). Assuming she can sell all of the lemonade she makes, find a formula for $R(g)$, the total amount of money (in dollars) Marisa takes in from lemonade sales, i.e. her revenue, if she makes g gallons of lemonade.

e. [3 points] What is the minimum number of gallons of lemonade Marisa needs to make in order not to lose money (that is, how much lemonade does she need to make to break even)?

39.

[10 points] Annie Ant and Greta Grasshopper are having a debate about how to spend their time during October. Annie says that she will spend a total of 12 hours each day gathering food and building her anthill. Let B be the number of cm^3 of anthill that Annie builds in October, and let $D = g(B)$ be the number of grams of food that she gathers in October.

Annie knows that g is a linear function. She is also able to determine that if she builds 500 cm^3 of her anthill in October, then she will gather a total of 1500 grams of food but that if she builds only 150 cm^3 of her anthill, then she will gather a total of 2300 grams of food in October.

a. [4 points] Find a formula for $g(B)$.

b. [6 points] Find and interpret the slope and horizontal intercept of the graph of $D = g(B)$ in the context of this problem. *For each interpretation, remember to use a complete sentence and include units.*

We live in a moment of history where change is so speeded up that we begin to see the present only when it is already disappearing.

- R. D. Laing