

(basic calculator permitted)

To obtain any credit, you must show your work for each problem! Place a box around each answer.

1. [4 pts] Let $P = (7, 11)$ and $Q = (2, -1)$. Find the *slope* of the line joining P and Q .

Solution: The slope of the line joining the points P and Q is:

$$m_{PQ} = \frac{\text{rise}}{\text{run}} = \frac{-1 - 11}{2 - 7} = \frac{-12}{-5} = \frac{12}{5}$$

2. Albertine buys an old castle in GammaVille. At time $t = 0$, there are 1593 bats and 4173 rats. Suppose that the bat population grows linearly at a rate of 55 bats per year and that the rat population declines linearly at the rate of 42 rats per year.

(a) [3 pts] Write an equation for the number of bats $B(t)$ at time t .



Solution: $B(t) = 1593 + 55t$

(b) [3 pts] Write an equation for the number of rats $R(t)$ at time t .



Solution: $R(t) = 4173 - 42t$

3. [1 pt each] Consider the straight line $y = x - 1$. For each of the following points in the plane determine whether

- the point lies on the line;
- the point lies above the line, or
- the point lies below the line.

Write: **ON**, **ABOVE**, or **BELOW** next to each point below.

(a) (2, 0)

(b) (4.3, 3.3)

(c) (-3, -4)

(d) (1, -2)

Solution: Let's call this function $y = f(x)$.

(a) (2, 0)

Now $f(2) = 2 - 1 = 1 > 0$. Thus (2, 0) lies **BELOW** the line.

(b) (4.3, 3.3)

Now $f(4.3) = 4.3 - 1 = 3.3$. Thus (4.3, 3.3) lies **ON** the line.

(c) (3, 2)

Now $f(3) = 3 - 1 = 2$. Thus (3, 2) lies **ON** the line.

(d) (1, -2)

Now $f(1) = 1 - 1 = 0 > -2$. Thus (1, -2) lies **BELOW** the line.

4. [2 pts each] Which of the following functions might be linear? Explain.

(a)

| t | 1 | 2 | 3 | 4 | 5 |
|------|----|----|----|---|---|
| G(t) | 19 | 15 | 11 | 7 | 3 |

Answer: This appears to be a **linear function**.

Explanation: Examining slopes between consecutive points

The slope between (1, 19) and (2, 15) is $\frac{\text{rise}}{\text{run}} = \frac{15-19}{2-1} = -4$

The slope between (2, 15) and (3, 11) is $\frac{\text{rise}}{\text{run}} = \frac{11-15}{3-2} = -4$

The slope between (3, 11) and (4, 7) is $\frac{\text{rise}}{\text{run}} = \frac{7-11}{4-3} = -4$

The slope between (4, 7) and (5, 3) is $\frac{\text{rise}}{\text{run}} = \frac{3-7}{5-4} = -4$

(b)

| x | 0 | 5 | 10 | 15 |
|------|---|----|----|-----|
| F(x) | 0 | 45 | 90 | 105 |

Answer: **Non-linear**

Explanation: The slope between (0, 0) and (5, 45) is $\frac{\text{rise}}{\text{run}} = \frac{45-0}{5-0} = 9$.

However, the slope between (10, 90) and (15, 105) is $\frac{\text{rise}}{\text{run}} = \frac{105-90}{15-10} = \frac{15}{5} = 3$

(c)

| x | 0 | 100 | 300 | 600 |
|------|----|-----|-----|-----|
| g(x) | 20 | 120 | 320 | 620 |

Answer: This appears to be a **linear function**.

Explanation: Examining slopes between consecutive points.

The slope between (0, 20) and (100, 120) is $\frac{\text{rise}}{\text{run}} = \frac{120-20}{100-0} = \frac{100}{100} = 1$

The slope between (100, 120) and (300, 320) is $\frac{\text{rise}}{\text{run}} = \frac{320-120}{300-100} = \frac{200}{100} = 1$

The slope between (300, 320) and (600, 620) is $\frac{\text{rise}}{\text{run}} = \frac{600-300}{620-320} = 1$

Extra credit: [4 pts] Consider the three points: A = (1, 3), B = (-1, 2), and C = (4, 5). Can these points lie on the same straight line? Explain!

Solution: We will compute the slope of the line joining A and B; then compute the slope of the line joining B and C.

$$m_{AB} = \frac{\text{rise}}{\text{run}} = \frac{2 - 3}{-1 - 1} = \frac{1}{2}$$

$$m_{BC} = \frac{\text{rise}}{\text{run}} = \frac{5 - 2}{4 - (-1)} = \frac{3}{5}$$

Since these two slopes are different, the points A, B, C cannot be collinear.

There is a very fine line between loving life and being greedy for it.

– Maya Angelou