To earn full credit, please show your reasoning.

1. *[4 pts]* Find the solution to the following linear system by using your method of choice. Give both coordinates of the solution:

$$2x + 4y = -14$$

$$y = 6x - 23$$

Solution: *Rewrite the second equation in standard form:* $6x - y = 23$

Multiply the first equation by -3:

$$-6x - 12y = 42$$

Now, to solve the system

$$6x + 12y = -42 \text{ and}$$

$$-6x + y = -23$$

we add the two equations to obtain:

$$13y = -65$$

And so $y = -5$

Hence $\boldsymbol{x = 3}$.

Substituting $x = 3$ *in either equation yields* $\boldsymbol{y = -5}$.

2. *[4 pts]* Using the method of Gaussian elimination, solve the following system. Give both coordinates of the solution:

$$x + 3y = -3$$

$$4x - 5y = 22$$

Solution: *Multiply the first equation by -4 to obtain the new system:*

$$-4x - 12y = 12$$

$$4x - 5y = 22$$

Adding yields: $-17y = 34$.

Hence $\boldsymbol{y = -2}$. *Substituting in either equation above yields* $\boldsymbol{x = 3}$.

3. *[4 pts]* Using the method of Gaussian elimination, solve the following system. Give both coordinates of the solution:

$$7x + 8y = 37$$

$$10x - 3y = 125$$

Solution: *Multiply the first equation by 10 and the second equation by -7 to obtain the system:*

$$70x + 80y = 370$$

$$-70x + 21y = -875$$

Adding yields: $101y = -505$. *Hence* $\boldsymbol{y = -5}$. *Substituting in either equation yields* $\boldsymbol{x = 11}$.

4. *[3 pts]* Give an *example* of a system (in two unknowns) that is inconsistent.

Answer: 3x + y = 1 and 3x + y = 3

5. *[4 pts]* Solve the following problem by introducing two variables and then solving. *Do not forget to introduce your variables! Little or no credit will be given for guessing.*

The sum of the ages of Jack and Jill is 61 years. Twenty years from now Jill will be 26 years less than twice Jack's age. Find their current ages.

Solution: *Let x = Jack's age now, and let y = Jill's age now.*

Twenty years from now Jack will be x+20 years old, and Jill will be y+20 years old.

The sum of their current ages is 61: x + y = 61

Twenty years from now Jill will be 26 years *less* than twice Jack's age.

Thus y + 20 = 2(x + 20) − 26.

Simplifying: y + 20 = 2x + 40 − 26 ⇒ $2x - y = 6$

Next, we solve the system:

x + y = 61

$2x - y = 6$.

Adding the two equations yields: 3x = 67. Thus x = 22.5. Since x + y = 61, y = 38.5.

Therefore, Jack currently is 22.5 years old, and Jill is 38.5 years old.

6. *[4 pts]* Establish the solution to the following problem using *two unknowns*, but ***do not solve****! Do not forget to introduce your variables!*

Albertine has $11.25 in quarters and dimes. If the number of dimes is 9 less than twice the number of quarters, how many coins of each type does she have?

Solution: *Let d = the number of dimes that Albertine has and let q = the number of quarters that Albertine has.*

We are given that ***d = 2q – 9****.*
Since each dime is worth 0.1 dollars, and each quarter is worth 0.25 dollars, we have:

0.1 d + 0.25 q = 11.25

One never reaches home, but wherever friendly paths intersect the whole world looks like home for a time.

- Hermann Hesse