*In the following set of problems, you obtain any credit, **you must show your work!***

1. Find the *greatest common factor* of the numbers a = 2205 and b = 2050.

Solution: *Using a factor tree: $2205 = (3^2)(5)(7^2)$ and $2050 = (2)(5^2)(41)$*

*Thus the greatest common factor (or GCD) of 2205 and 2050 is **5**.*

2. Simplify each o+f the following:

(a) $49^{-\frac{3}{2}}$

Solution: $49^{-\frac{3}{2}} = \frac{1}{49^{\frac{3}{2}}} = \frac{1}{\left(49^{\frac{1}{2}}\right)^3} = \frac{1}{7^3} = \mathbf{\frac{1}{343}}$

(b) $\left(100^{-\frac{1}{2}}\right)(125)^{\frac{1}{3}}$

Solution: $\left(100^{-\frac{1}{2}}\right)(125)^{\frac{1}{3}} = (10^2)^{-\frac{1}{2}}(5^3)^{\frac{1}{3}} = (10^{-1})5 = \frac{5}{10} = \mathbf{\frac{1}{2}}$

3. Express the following expression without using negative exponents. Simplify if possible.

$$\left(\frac{a^{-3}b^2c^{-4}}{b^{-5}}\right)^{-2}$$

Solution: $\left(\frac{a^{-3}b^2c^{-4}}{b^{-5}}\right)^{-2} = (a^{-3}b^7c^{-4})^{-2} = a^6b^{-14}c^8 = \mathbf{\frac{a^6c^8}{b^{14}}}$

4. *Simplify fully:* $\left(\frac{4}{9}\right)^{-\frac{1}{2}}$

Solution: $\left(\frac{4}{9}\right)^{-\frac{1}{2}} = \left(\frac{9}{4}\right)^{\frac{1}{2}} = \frac{9^{1/2}}{4^{1/2}} = \mathbf{\frac{3}{2}}$

5. *Simplify fully:* $\left(\frac{243}{32}\right)^{\frac{4}{5}}$

Solution: $\left(\frac{243}{32}\right)^{\frac{4}{5}} = \frac{243^{4/5}}{32^{4/5}} = \frac{\left(243^{1/5}\right)^4}{\left(32^{1/5}\right)^4} = \frac{(3)^4}{(2)^4} = \mathbf{\frac{81}{16}}$

6. Factor *fully:*

(a) $15a^9b^5c^3 - 25a^8b^3c + 35a^6b^2c^2$

Solution:

$15a^9b^5c^3 - 25a^8b^3c + 35a^6b^2c^2 = \mathbf{5a^6b^2c(3a^3b^3c^2 - 5a^2b + 7c)}$

(b) Factor $x^2 - 11x + 30$

Solution: $x^2 - 11x + 30$

$x^2 - 11x + 30 = (\boldsymbol{x - 6})(\boldsymbol{x - 5})$

(c) Factor $15z^4 - 25z^5$

Solution:

$15z^4 - 25z^5 = \boldsymbol{5z^4(3 - 5z)}$

(d) Factor $x^2 - x - 56$

Solution: $x^2 - x - 56 = (\boldsymbol{x - 8})(\boldsymbol{x + 7})$

7. A speck of dust in an electron microscope is 1.2×10^3 millimeters wide. The image is 5×10^6 times larger than the actual size. How many millimeters wide is the actual speck of dust?

Solution:

Let the actual size be y (millimeters).

Then $y(5 \times 10^6) = 1.2 \times 10^3$

Hence $y = \frac{1.2\times10^3}{5\times10^6} = \frac{1.2}{5\times10^3} = 0.24 \times 10^{-3} = \boldsymbol{2.4 \times 10^{-4}}$ ***millimeters***

8. *Express in scientific notation:* $\frac{7\times10^5}{2\cdot10^{-2}\times2.5\cdot10^9}$

Solution: $\frac{7\times10^5}{2\cdot10^{-2}\times2.5\cdot10^9} = \frac{7}{2(2.5)} \times 10^{5-(-2)-9} = \boldsymbol{1.4 \times 10^{-2}}$

EXTRA CREDIT RIDDLES *[1 pt each]:*

1. A hockey stick and ball together cost $50. If the stick costs $49 more than the ball, what is the cost of each?

Answer: The hockey stick costs $ 49.50 and the ball costs $ 0.50

2. There are two ducks in front of a duck, two ducks behind a duck and a duck in the middle. How many ducks are there? Explain.

Answer: 3 ducks is the smallest number possible. But any odd number greater than or equal to 3 works as well.

3. What starts with the letter "T," is filled with "T," and ends in "T?"

Answer: teapot

4. What is greater than God, more evil than the devil, the poor have it, the rich need it, and if you eat it, you will die?

Answer: nothing