MATH 100 SOLUTIONS: TEST I

Instructions: Choose any 14 of the following 17 problems. You may answer more than 15 to earn extra credit! *For each problem, be certain to show your work!* You may use your calculator, but you still must show your reasoning! Be sure to place a box around your answer.

To obtain any credit, you must show your work! Place a box around each answer.



1. Light leaving the Sun takes about 88 minutes to reach the Earth, traveling a distance of approximately 93,000,000 miles. Find the speed of light in miles per second. *Alert:* In this question, time is given in minutes, not seconds.

Solution: Conveting to seconds, 88 minutes = 60(88) = 5280 seconds. Thus speed = $\frac{distance}{time} = \frac{93000000}{5280} = 17,613.6$ miles/second

2. Consider the expression $\frac{6a-3b-ab+2a^2}{2a-b}-3$.

Evaluate this expression if a = 5 and b = 1. *Simplify* your answer.

Solution:
$$\frac{6a-3b-ab+2a^2}{2a-b} - 3 = \frac{6(5)-3(1)-(5)(1)+2(5)^2}{2(5)-1} - 3$$
$$= \frac{30-3-5+50}{9} - 3 = \frac{72}{9} - 3 = 5$$

3. Solve for *x* in the following equation:

179 - 18(x - 10) = 158 - 3(x - 17)

Solution: distributing on each side of the equation: 179 - 18x + 180 = 158 - 3x + 51 which implies 15x = 179 + 180 - 158 - 51 = 150 and so $x = \frac{150}{15} = 10$

4. The perimeter of a triangular garden is 99 meters. Find the length of each of the three sides if one side is 8 meters greater than *twice* the length of the smallest side, and the third side is 5 meters less than *three* times the length of the smallest side. (Set up the equation using one-variable; do not solve unless you are using more than one variable.)

Solution: Let x be the shortest side (in meters).

Then the second side is of length 2x + 8, and the third side is of length 3x - 5. Since the perimeter is 99 meters, we obtain the equation: x + (2x + 8) + (3x - 5) = 99

5. Simplify fully by removing brackets. *Show every step!* Circle your final answer.

 $8(b-c) - [-\{a-b-3(c-b+a)\}]$

Solution:

Expanding from the innermost pair of parentheses: $8(b-c) - [-\{a-b-3c+3b-3a\}]$ Simplifying: $8(b-c) - [-\{-2a+2b-3c\}]$ Distributing the negative sign: 8(b-c) - [2a-2b+3c]Distributing the second negative sign: 8(b-c) - 2a + 2b - 3cSo 8b - 8c - 2a + 2b - 3cFinally, gathering like terms: -2a + 10b - 11c

6. If Albertine was *x* years old *y* years ago, *how old* will Albertine be *z* years from today? (Express your answer in terms of the constants *x*, *y*, and *z*.)

Solution: Since Albertine was x years old y years ago, today she is x + y years old. Now, z years from now she will be x + y + z years old.

7. Consider the following geometric figure. Assume that units are in cm.

(a) Find the perimeter (use appropriate units). Solution: The perimeter is: 1 + 1 + 9 + 4 + 10 + 5 = 30 cm

(b) Find the area (use appropriate units). Solution: The area is $36 + 5 = 41 \text{ cm}^2$



8. If avocados sell for x dollars per dozen and grapefruits sell for y dollars per half-dozen, how much (in dollars) will it cost to buy 3 avocados and 5 grapefruits

Solution: Each avocado sells for x/12 dollars, and each grapefruit sells for y/6 dollars. Thus 3 avocados and 5 grapefruits cost

$$3(x/12) + 5(y/6) = x/4 + 5y/6$$
 dollars

9. Subtract $2x^4 - 9x^3 + 2x^2 - 3x + 4$ from $5x^4 + 11x^3 - x^2 + 4x - 4$ and simplify your result. Show your work!

Solution: $5x^4 + 11x^3 - x^2 + 4x - 4 - (2x^4 - 9x^3 + 2x^2 - 3x + 4) =$ $5x^4 + 11x^3 - x^2 + 4x - 4 - 2x^4 + 9x^3 - 2x^2 + 3x - 4 =$ $3x^4 + 20 x^3 - 3x^2 + 7x - 8$

10. Find the sum of 5a - 9b + c and 3b - 9c, and subtract the result from 11a - 9b + c. Simplify your answer.

Solution: Sum = (5a - 9b + c) + (3b - 9c) = 5a - 6b - 8cSubtracting the sum from 11a - 9b + c, 11a - 9b + c - (5a - 6b - 8c) = 11a - 6b + c - 5a + 6b + 8c = 6a - 3b + 9c

11. Jack and Jill went shopping to purchase a tent. Patagonia had a sale in which the price of every tent is reduced by 27%. They selected a tent and paid \$234.56. Find the *original* cost of the tent.

Solution: Let x = original cost of tent (in dollars).

Then, the sale price is 0.73 x. Thus 0.73x = 234.56, from which we obtain x = 234.56/0.73 = \$321.32

12. Albertine works in a biomedical engineering lab. She requires a 15% acid solution for a critical test, but her supplier only ships a 10% solution and a 30% solution. Rather than pay the hefty surcharge to have the supplier make a 15% solution, she decides to mix 10% solution with 30% solution, to make her own 15% solution. Albertine needs 10 liters of the 15% acid solution. How many liters of 10% solution and 30% solution should she use? (Set up the equations. Do not solve unless you choose to use two variables.

Solution: Let x be the number of liters of the 10 % solution that is needed. Then 10 - x is the number of required liters of the 30 % solution.

Thus 0.1x + 0.3(10 - x) = 0.15(10)

13. One number exceeds another by 48, and their sum is 4086; find the smaller number. (You need not solve unless you are using more than one variable.) Guessing will receive little or no credit.

Solution: Let x be the smaller number. Then the larger number is x + 48. So our equation is: x + (x + 48) = 4086

14. Solve each of the following equations:

(a) |x-5|=3

Solution: Either x - 5 = 3 OR x - 5 = -3. Solving the first equation: x = 8. Solving the seond equation: x = 2. Next, you should check your answers by substitution.

(b) 4 |1 - 2x| = 20
Solution: Dividing by 4, we obtain |1 - 2x| = 5.
So either 1 - 2x = 5 OR 1 - 2x = -5.
Solving the first equation: x = -2.
Solving the seond equation: x = 3.
Next, you should check your answers by substitution.

(c) |5x + 2019| = -1789

Solution: Since the absolute value of a number can never be negative, this equation has no solution!

15. Find four consecutive *even* numbers whose sum is 8108. Guessing will earn minimal credit. Introduce your variable; you need not solve unless you choose to use more than 1 variable.

Solution: Let n = smallest of the four integers. Then the others are n + 2, n + 4, and n + 6. Thus n + (n + 2) + (n + 4) + (n + 6) = 8108So 4n + 12 = 8108

16. The town of LostVille is experiencing a catastrophic event due to the degradation of air quality and drinking water.

Now there are 1234 residents of LostVille. Next year the town will lose 34% of its population. The following year the town will lose 33% of those left. During the third year, the town will lose 19% of its inhabitants. Find the population of LostVille after 3 years have passed.

Solution: After the first year, there were (0.66)(1234) residents.

After the next year, there were (0.67)(0.66)(1234) residents.

After the third year, there were (0.81)(0.67)(0.66)(1234) = (approximately) 442 residents.



17. Albertine can drive her Tesla 120 miles on a mountain road in the same amount of time it takes her to drive 156 miles on the freeway. Find her freeway speed if she can drive 15 miles per hour faster on the freeway than she can drive on a mountain road. Introduce your variable; you need not solve unless you choose to use more than 1 variable.

Solution: Let s = Albertine's highway speed (in mph). Then her mountain road speed is s - 15 mph Since (speed)(time) = distance, we know that Albertine's freeway time = $\frac{156}{s}$ hours. Also her mountain road time = $\frac{120}{s-15}$ hours.

Since we are told that the two times are the same we obtain the equation:

$$\frac{156}{s} = \frac{120}{s-15}$$