

MATH 117 PRACTICE TEST III 5 APRIL 2018

The limits of my language mean the limits of my world.

- Ludwig Wittgenstein, *Tractatus Logico-Philosophicus* (1922)

1. Find the *domain* of the functions

(a) $f(x) = 2016x + \sqrt{x^2 + x + 11}$

(b) $f(x) = \frac{x(x-3)(x-5)}{x+99}$

2. Show that $x = 1$ is a root of the polynomial $p(x) = x^3 - 4x^2 - 13x + 16$.

Then find the quotient when $p(x)$ is divided by $x - 1$.

3. Consider the function $g(x) = x^4 - 7x^2 - 6x$. Circle the numbers below that are roots of g . (There may be several or perhaps none.) Show your work.

(a) 0 (b) 1 (c) -1 (d) 2 (e) -2 (f) 3 (g) -3 (h) 7

4. Divide $p(x) = x^6 - 3x^5 + 2x^4 - x + 13$ by $x^2 - x + 3$.

Quotient = _____

Remainder = _____

5. Let $g(x) = 2x^2 + x + 1$. Find and *simplify* the expression

$$\frac{g(2+h) - g(2)}{h}$$

6. [University of Michigan precalculus final exam] In this problem, the constants a , b , c , and d are all positive and different from each other. Consider the function:

$$G(x) = \frac{x^2(x+3)}{(x^2+9)(x+13)}$$

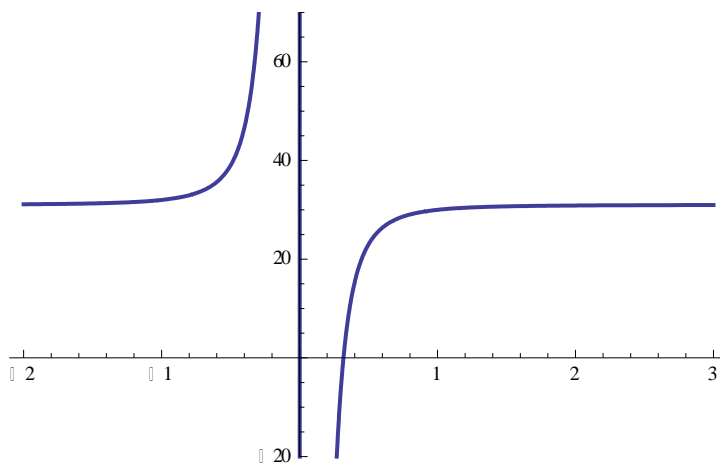
- (a) What is the y-intercept of G ? If there is not a y-intercept, write *NONE*.
(b) Find all zeroes of G . If there are no zeroes, write *NONE*.
(c) Give the equation(s) of all vertical asymptote(s) of G . If there are none, write *NONE*.

7. Find the *domain* of each of the following functions. *Explain!*

$$(a) \quad y = \frac{(x+5)(x-7)(x+8)(x^2+4)}{(3x-11)(x-1)} + x^3 + 1$$

$$(b) \quad y = 17 + 79\sqrt{\frac{x+11}{8-x}}$$

8. Let $F(x) = 31 - \frac{1}{x^3}$. Here is a plot of F .



- What is the *domain* of F ?
- What are the vertical asymptote(s)?
- Find any horizontal asymptote(s).
- What is the *range* of F ?

9. Suppose that $h(x) = 1 + \sqrt{\frac{x+3}{x+1} + 9}$

(a) If $f(x) = 1 + \sqrt{x+9}$

find a function g such that $h = f \circ g$

(b) If $f(x) = 1 + \sqrt{x}$

find a function g such that $h = f \circ g$

10. If $g(x) = \sqrt{\frac{x+3}{x+1} + 9}$

find a function f such that $h = f \circ g$

11. . Suppose that $h(x) = \sqrt{\frac{4}{x^2 + 7} + 13}$

12. If $f(x) = \sqrt{x}$ find a function g such that $h = f \circ g$

13. If $f(x) = \sqrt{x + 13}$ find a function g such that $h = f \circ g$

14. If $g(x) = \frac{1}{x^2 + 7}$ find a function f such that $h = f \circ g$

15. How many real roots does each of the following equations possess? (Hint: These questions require very little calculation.)

(a) $(x - 1)(x + 5)(x^2 + 13)^4 = 0$

(b) $5x^2 - 4x + 1 = 0$

(c) $x^2 - 4x - 1 = 0$

(d) $3x^2 - 4x + 8 = 0$

(e) $(x^4 + 2)(x + 1)(x^2 - 9) = 0$

16. Find functions f and g , each simpler than the given function h , such that $h = f \circ g$.

(a) $h(x) = \sqrt{2x + 1}$

(b) $h(x) = (x^4 + 4)^{13}$

(c) $h(x) = \frac{5}{x + 99}$

(d) $h(x) = \sqrt{\frac{x + 4}{x + 3}}$

17. Factor fully each of the following:

(a) $x^3 + x^2 - 7x + 2$ given that $x = 2$ is a root

(b) $x^4 + 6x^3 - 68x^2 - 150x - 77$ given that $x = -1$ is a root of multiplicity 2

(c) $x^3 - 8$ given that $x = 2$ is a root

(d) $x^5 + 1$ given that $x = -1$ is a root

18. Find an equation of a polynomial that has zeroes at $x = 1, 4, 5$ and has y-intercept of 11.

19. (a) Without actually dividing, explain why $x + 3$ is a factor of $x^3 + 6x^2 + 11x + 6$

(b) Explain why $x - 1$ is a factor of $32x^{74} - 33x^{33} + 1$

20. Multiply the following two polynomials:

$$p(x) = 3x^3 - 4x^2 + x + 1$$

$$q(x) = x^5 + x^3 + 3x^2 + 1$$

21. Consider the polynomial

$$y = f(x) = -x^2(x - 2)^4(x - 3)^5(x - 5)(x^2 + x + 1)$$

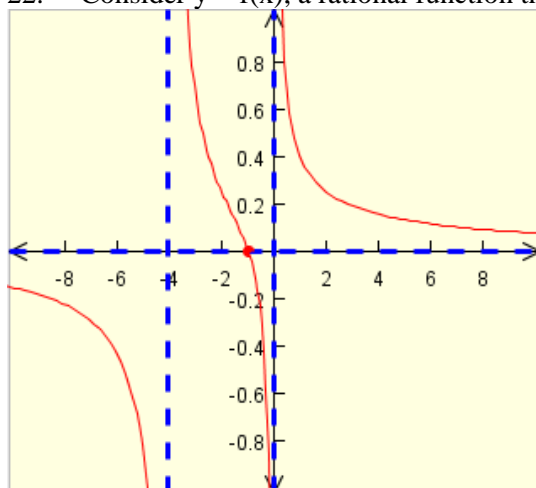
(a) The domain of f is:

(b) The zeroes of the polynomial are:

(c) What happens to y as $x \rightarrow \infty$?

(d) What happens to y as $x \rightarrow -\infty$?

22. Consider $y = f(x)$, a rational function that has the graph below:



(a) List the zeroes.

(b) List the singularities.

(c) List the horizontal asymptotes.

(d) List the vertical asymptotes.

23. Perform polynomial division. What is the quotient? What is the remainder?

$$(a) \frac{x^3 - 3x^2 + x + 4}{x + 1}$$

$$(b) \frac{x^3 - x^2 + x + 4}{x^2 + x + 1}$$

24. Find the quotient when $x^3 - 4x^2 + 5x + 6$ is *divided* by $x - 2$.

25. What are the roots of the polynomial $p(x) = x^3 - 2x^2 - 23x + 24$?
Hint: Since $p(1) = 0$, $x = 1$ is a root of this polynomial.

26. For which value(s) of c will the following polynomial be divisible by $x - 3$?

$$P(x) = 3x^4 - cx^2 + 3x - 72$$

27. Consider the following two rational functions:

$$r(x) = \frac{x}{x^2 + 4} \quad \text{and} \quad s(x) = \frac{2}{2x - 3}$$

Express each of the following as a rational function. Simplify.

- (a) $r(x) + 3s(x)$
- (b) $(s(x))^2$
- (c) $(rs)(x)$
- (d) $r \circ s(x)$
- (e) $s \circ r(x)$
28. Find a polynomial $p(x)$ that has roots 1, 2, -3, -4 and satisfies the property that $p(-1) = 5$.
29. Explain the significance of the *discriminant of a quadratic expression* $Ax^2 + Bx + C$. Give examples of each of the three *types* of discriminants and their relationship to the corresponding graph of the parabola.
30. *How many real roots* does each of the following equations possess? (*Hint*: These questions require very little calculation.)
- (f) $(x - 1)(x + 5)(x^2 + 13)^4 = 0$
- (g) $5x^2 - 4x + 1 = 0$
- (h) $x^2 - 4x - 1 = 0$
- (i) $3x^2 - 4x + 8 = 0$
- (j) $(x^4 + 2)(x + 1)(x^2 - 9) = 0$
31. Let $p(x) = x^2 + x - 2$ and $q(x) = x^2 + 1$. Compute and simplify each of the following:
- (a) $4p(x)$
- (b) $(p + q)(x)$
- (c) $(p - q)(x)$
- (d) $(3p + 5q)(x)$
- (e) $(pq)(x)$
- (f) $p \circ q(x)$
- (g) $q \circ p(x)$
- (h) $q \circ q(x)$
- (i) $p(x + 1) - p(x)$
32. If p and q are polynomials, in general, what can be said of $\deg(p + q)$, $\deg(p - q)$, $\deg(pq)$

33. What is meant by the “dominant term” of a polynomial? What does this term tell us about the graph of the polynomial?

34. What is meant by “zero” of a polynomial?

35. Let $p(x) = (x - 2)^2(x^2 + x + 1)^3(x^2 - 14x + 45)$

(a) What is the degree of p ?

(b) Find all the zeroes of p .

36. Graph each of the following polynomials. Focus primarily upon the qualitative behavior of the graph instead of plotting points.

(a) $F(x) = 2x + 5$

(b) $y = x^2 - 9$

(c) $G(x) = 1 + x^4$

(d) $h(x) = (x + 1)^3 + 2$

(e) $q(x) = x^2(x - 1)^2$

(f) $F(x) = (x + 2)(x + 3)(x + 4)$

37. Find the zeroes of the polynomial $f(x) = x^6 - 8x^3 + 15$. Hint: Let $z = x^3$; then solve for z .

38. Find all zeroes of the polynomial $g(x) = x^8 - 1$ Hint: Factor as difference of squares several times.

39. Find a number b such that 4 is a zero of the polynomial

$$p(x) = x^3 - 2x^2 + bx + 1.$$

40. Find a polynomial of third degree having roots (i.e., zeroes) of 1, 7, 11.

41. Let $p(x) = x^3 - 2x^2 + x$. For each given point, determine if it lies *on* the graph, *above* the graph, or *below* the graph of $y = p(x)$.

(a) (1, 1), (b) (2, 1), (c) (3, 12), (d) (-1, -5)

42. Which (if any) of the following polynomials have $x - 1$ as a factor?

(a) $x^9 - 1$

(b) $x^6 - x^3 + 4x^2 - 13x + 9$

(c) $2x^{13} - 4x + 3$

(d) $x^{2014} - 5x + 4$

43. For each of the following polynomials find at least one linear factor. *Hint:* Use the Factor Theorem.

(a) $x^3 - 4x - 3$

(b) $x^4 - 9x + 2$

(c) $x^3 - 2x^2 + 3x - 18$

- (d) $x^5 + 32$
 (e) $x^{2018} - b^{2018}$

44. (a) For which value(s) of c will the following polynomial be divisible by $x - 2$?

$$p(x) = x^3 - cx^2 + 5x - 1$$

(b) For which value(s) of c will the following polynomial be divisible by $x + 3$?

$$p(x) = x^3 + 5x^2 - 2x + c$$

45. Using the factor theorem, factor fully: $x^3 - 1$ and $x^3 + 1$.

46. Long division practice.

(a) $\frac{x^3 + x^2 + x + 4}{x + 1}$

(b) $\frac{x^4 + 5x + 4}{x^2 + 2}$

(c) $\frac{x^4 + x^2 - x + 5}{x^4 + 2}$

(d) $\frac{2x^5 - x^3 - 1}{2x^2 - 1}$

(e) $\frac{x^5 - 1}{x - 1}$

(f) $\frac{x^4}{x + 2}$

47. Find the *domain* of each of the rational functions in question 46.

48. What is meant by *singularity* of a rational function?

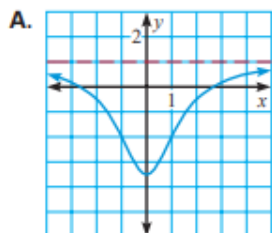
50. Let $R(x) = \frac{5x^8 + x^7 + 14x + 21}{3x^8 + 33x^6 - 5x^4 + x + 11}$. What is the approximate value of $R(10^{10})$?

What about $R(-10^{10})$?

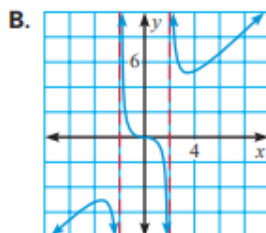
51.

MATCHING GRAPHS Match the function with its graph.

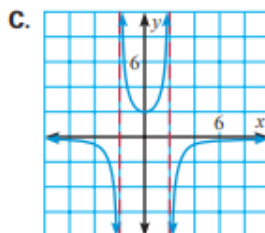
20. $y = \frac{x^2 - 7}{x^2 + 2}$



21. $y = \frac{-8}{x^2 - 4}$

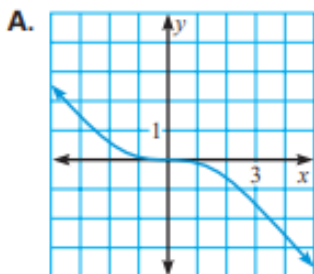


22. $y = \frac{x^3}{x^2 - 4}$

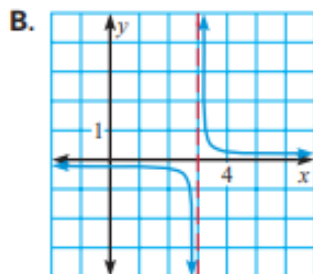


52. Match

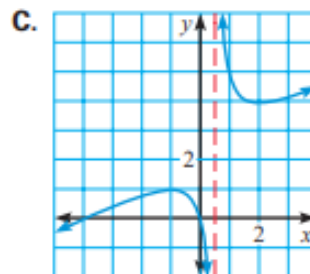
23. $y = \frac{3}{x^3 - 27}$



24. $y = \frac{-x^3}{x^2 + 9}$



25. $y = \frac{x^2 + 4x}{2x - 1}$



53. Match.

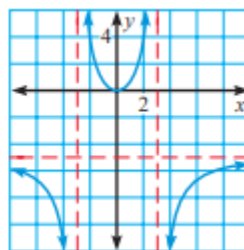
MULTIPLE CHOICE Which of the following functions is graphed?

(A) $y = \frac{-5x^2}{x^2 + 9}$

(B) $y = \frac{5x^2}{x^2 - 9}$

(C) $y = \frac{5x^2}{x^2 + 9}$

(D) $y = \frac{-5x^2}{x^2 - 9}$



54. For each of the following functions, determine *domain*, *zeroes*, *singularities*, *limiting behavior* for large $|x|$, and perform a *sign analysis*. Sketch the curve indicating any and all horizontal and vertical asymptotes.

(a) $y = (x - 1)(x - 2)(x - 3)$

(b) $y = (x - 1)^2(x - 2)^3(x - 3)$

(c) $y = x^4(x + 5)^2(x - 3)^4$

(d) $y = \frac{1}{x + 3}$

(e) $y = \frac{x - 2}{x + 1}$

(f) $y = \frac{x - 2}{x^2 + 1}$

(g) $y = \frac{(x - 1)(x - 2)}{x - 3}$

$$(h) y = \frac{x(2x-1)^2(x-2)}{(x-3)^3}$$

$$(i) y = \frac{(x^2+1)(x-6)(x-8)^2}{(x-3)(x-9)}$$

$$(j) y = \frac{1}{(x-1)(x+2)(x-4)}$$

$$(k) y = \frac{(x+5)^4(x-2)^6}{(x-1)^9}$$

55.

Find the vertical asymptotes, horizontal asymptote, domain and range of the following functions.

$$7. f(x) = \frac{12}{x+7} - 8 \qquad 8. f(x) = \frac{-3x+2}{-4x-7}$$

$$9. f(x) = \frac{x-1}{x+5}$$

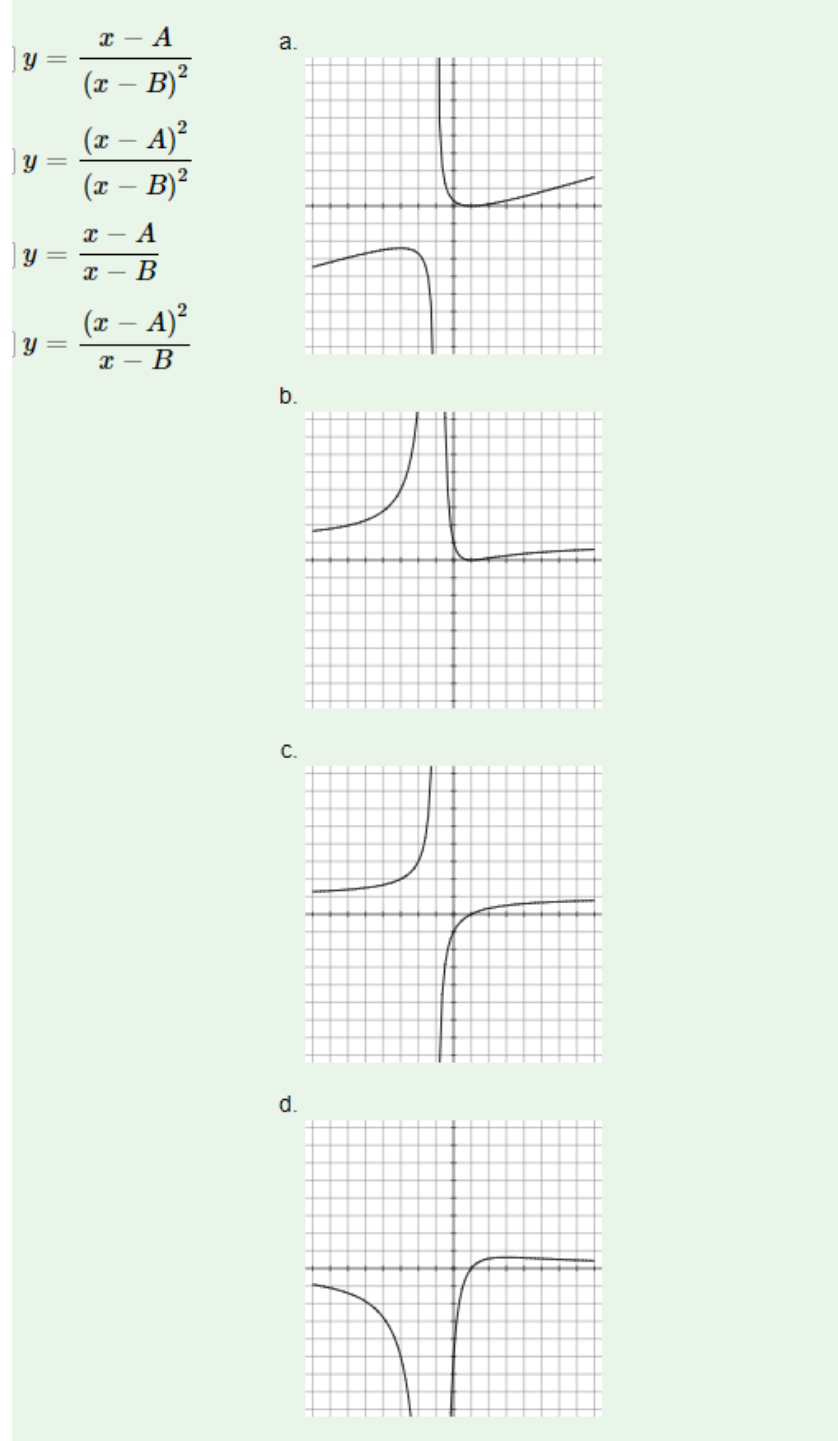
56.

Sketch a graph of $f(x) = \frac{(x+2)(x-3)}{(x+1)^2(x-2)}$.

57.

Given the function $f(x) = \frac{(x+2)^2(x-2)}{2(x-1)^2(x-3)}$, use the characteristics of polynomials and rational functions to describe its behavior and sketch the function.

58. Match.



You propound a complicated arithmetical problem: say cubing a number containing four digits. Give me a slate and half an hour's time, and I can produce a wrong answer.

- George Bernard Shaw