Math 115 - Team Homework Assignment #2, Winter 2016

- **Due Date:** January 27 or 28 (Your instructor will tell you the exact date and time.)
- Note: All problem, section, and page references are to the course textbook, which is the 6th edition of *Calculus: Single Variable* by Hughes-Hallett, Gleason, McCallum, et al.
- Remember to follow the guidelines from the "Doing Team Homework" and "Team HW Tutorial" links in the sidebar of the course website.
- Do not forget to rotate roles and include a reporter's page each week.
- Show ALL your work.

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1. Shelly is an entrepreneur, and she sells seashells by the seashore. She has found that her business varies based on the time of year. Her sales are lowest, at 10 shells a day, on February 1, and her sales are highest, at 90 shells a day, on August 1. Let $S(t)$ be a sinusoidal function modeling the number of shells Shelly sells a day t months after January 1, 2014.

   (a) Sketch the function $S(t)$ over the domain $0 \leq t \leq 24$.

   (b) Write two formulas for $S(t)$, one using a sine function, and one using a cosine function.

   (c) Suppose Shelly is happy when she sells at least 30 shells a day, and unhappy otherwise. For how many months between January 1, 2014 and January 1, 2016 was Shelly happy? (Solve this problem algebraically, and be sure to show your work carefully. Otherwise, Shelly will get angry, and you wouldn't like Shelly when she's angry. :-))

2. For each of parts (a)-(f) below, you are asked to find the formula of a function satisfying certain properties. It is possible that there are many functions satisfying those properties, and it is also possible that there are no functions satisfying those properties. In each case, find a formula for one specific function, or, if no such functions exist, explain why.
   For example, if we were to ask for a linear function $f(x)$ such that the graph of $y = f(x)$ has slope 2, then both $f(x) = 2x + 42$ and $f(x) = 2x - 42\pi$ would be acceptable answers.

   Find a formula for:

   (a) a rational function $g(x)$, such that the graph of $y = g(x)$ has a horizontal asymptote at $y = 2$ and vertical asymptotes at $x = 1$ and $x = 3$.

   (b) a periodic function $h(w)$ of the form $h(w) = \ell(w)\sin(w)$, where $\ell(w)$ is a non-constant linear function.

   (c) a function $R(z)$ of the form $R(z) = \frac{S(z)}{T(z)}$, where $S(z)$ is an exponential function, $T(z)$ is a linear function, $R(z)$ has a vertical asymptote at $z = 2$, and $\lim_{z\to\infty} R(z) = 0$.

   (d) a polynomial $p(x)$ with $p(0) = 1$ and $\lim_{x\to\infty} p(x) = 0$.

   (e) a sinusoidal function $c(y)$ with a maximum value of 10, a minimum value of 0, $c(2) = 5$, and a period of 3.

   (f) a function $A(x)$ of the form $A(x) = \ln\left(\frac{B(x)}{C(x)}\right)$, where $B(x)$ and $C(x)$ are linear functions, $A(0) = 1$, and $\lim_{x\to\infty} A(x) = 1$.

3. Let $q(x) = \ln\left(\frac{x^2-x-6}{x-3}\right)$.

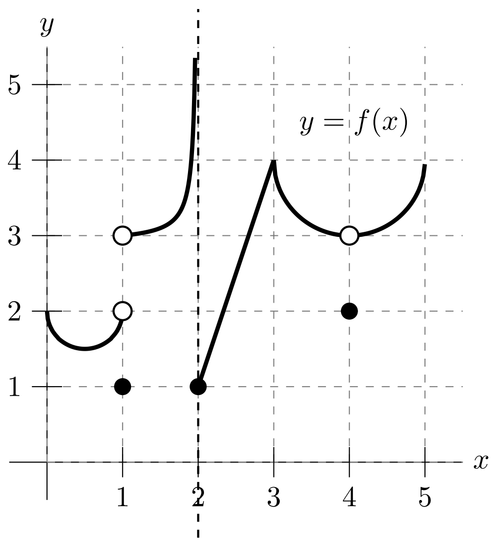
   (a) On what domain is $q(x)$ defined?

   (b) Does $\lim_{x\to 3} q(x)$ exist? If so, what is its value? Investigate numerically and then verify your answer algebraically.

   (c) Let $r(u) = q(\sin(u))$. On what domain is $r(u)$ defined? Explain your answer.

   (d) Let $s(v) = q\left(6e^{-2v}\right)$. On what domain is $s(v)$ defined? Explain your answer.

4. A portion of the graph of a function f is shown below:



   (a) Give all values c in the interval $0 < c < 5$ for which $\lim_{x\to c} f(x)$ does not exist.

   (b) Give all values c in the interval $0 < c < 5$ for which $\lim_{x\to c^+} f(x)$ does not exist.

   (c) Give all values c in the interval $0 < c < 10$ for which $f(x)$ is not continuous at c.

   (d) With f as shown in the graph above, define a function g by the formula

$$g(x) = \begin{cases} f(x) & \text{if } 0 < x < 5 \\ \dfrac{|4x^3 + A|}{Bx^3 + C} & \text{if } x \leq 0 \end{cases}$$

   where A, B, and C are constants.

   Find values of A, B, and C so that all of the following conditions hold:

   - $g(x)$ is continuous at $x = 0$.
   - $g(x)$ has a vertical asymptote at $x = -3$.
   - $\lim_{x\to -\infty} g(x) = -2$.

   Be sure to show your work.