Math 115 - Team Homework Assignment #3, Winter 2016

- **Due Date:** February 2 or 3 (Your instructor will tell you the exact date and time.)
- Note: All problem, section, and page references are to the course textbook, which is the 6th edition of *Calculus: Single Variable* by Hughes-Hallett, Gleason, McCallum, et al.
- Remember to follow the guidelines from the "Doing Team Homework" and "Team HW Tutorial" links in the sidebar of the course website.
- Do not forget to rotate roles and include a reporter's page each week.
- Show ALL your work.

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1. Charlie is running back and forth in a straight line between point A and point B. His distance from point B t seconds after he begins his workout is $C(t) = 14\cos\left(\frac{\pi}{8}t\right) + 14$ meters.

   (a) Sketch a graph of $C(t)$ for $0 \le t \le 40$.

   (b) Using your graph, when is Charlie's instantaneous velocity equal to 0 in the interval $0 < t < 40$?

   (c) What is the distance between point A and point B?

   (d) How long does it take for Charlie to run from point A to point B?

   (e) What is Charlie's average velocity during the first 16 seconds of his workout?

   (f) What is Charlie's average speed during the first 16 seconds of his workout?
   Remember that the **average speed** of an object over an interval of time is given by

   $$\text{Average speed} = \frac{\text{Distance travelled}}{\text{Time elapsed}}$$

   (g) Write an expression involving a limit that gives Charlie's instantaneous velocity 2 seconds after his workout begins. Do NOT evaluate the limit.

2. The table below gives several values of a function $w(x)$.

x	3.5	3.9	3.99	4	4.01	4.1	4.5
$w(x)$	7.091	7.818	7.982	8	8.030	8.309	9.586

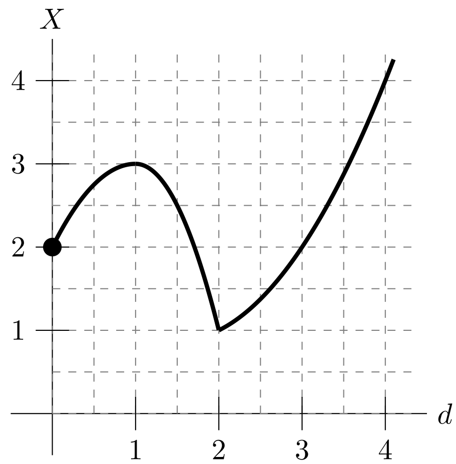
   Let $t(x) = \dfrac{w(x) - 8}{x - 4}$. Suppose $\lim\limits_{x\to 4^-} t(x)$ and $\lim\limits_{x\to 4^+} t(x)$ both exist.

   (a) Use the information in the table to estimate $\lim\limits_{x\to 4^-} t(x)$.

   (b) Use the information in the table to estimate $\lim\limits_{x\to 4^+} t(x)$.

   (c) Based on your answers above, do you expect $\lim\limits_{x\to 4} t(x)$ to exist? Explain why or why not.

3. In Townsville, USA, a vat of Chemical X is spilled into Lake Townsville, and Professor Utonium is sent to investigate. Let $c(d)$ be the concentration of Chemical X (in mg/L) at a depth of d meters below the surface in Lake Townsville. A portion of the graph of $X = c(d)$ is shown below.



(a) What is the concentration of Chemical X at the surface of Lake Townsville?

(b) On the domain $0 < d < 4$, over what intervals in $c'(d)$ positive?

(c) What is the average rate of change of the concentration of Chemical X over the interval from $d = 1$ to $d = 3$? Remember to include units.

(d) Suppose $c'(3) = A$. Estimate the value of A, and, using your answer, give a practical interpretation of the equation $c'(3) = A$ in the context of this problem. Remember to use a complete sentence and include units.

4. A plastic bead, initially at a temperature of $70\,^\circ$F is placed in a freezer, which is set to a constant temperature of exactly $-2\,^\circ$F. Let $p(t)$ be the temperature (in $^\circ$F) of the plastic bead at time t minutes after it is placed in the freezer. Assume that the plastic brick never reaches a temperature of exactly $-2\,^\circ$F and that the function p is differentiable.

(a) Why is it reasonable to assume that the function p is invertible?

For each of parts (b)-(d) below, remember to use a complete sentence and include units.

(b) Give a practical interpretation of the equation $p(10) = 55$ in the context of this problem.

(c) In the context of this problem, give a practical interpretation of the equation $p'(5) = -2$ that can be understood by someone who knows no calculus.

(d) Assume p^{-1} is also differentiable. In the context of this problem, give a practical interpretation of the equation $(p^{-1})'(8) = -7$ that can be understood by someone who knows no calculus.