Math 115 - Team Homework Assignment #4, Winter 2016

- **Due Date:** February 23 or 24 (Your instructor will tell you the exact date and time.)
- Note: All problem, section, and page references are to the course textbook, which is the 6th edition of *Calculus: Single Variable* by Hughes-Hallett, Gleason, McCallum, et al.
- Remember to follow the guidelines from the "Doing Team Homework" and "Team HW Tutorial" links in the sidebar of the course website.
- Do not forget to rotate roles and include a reporter's page each week.
- Show ALL your work.

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1. Some values of differentiable functions f and g and their derivatives are shown in the table below.

t	1	3	5
$f(t)$	2	5	-1
$f'(t)$	3	1	-2
$g(t)$	6	9	13
$g'(t)$	0.5	2	4

Evaluate each of the following derivatives.

(a) $\left.\dfrac{d}{dt}\left(f(t)+2g(t)\right)\right|_{t=5}$

(b) $\left.\dfrac{d}{dt}\left(tg(t)\right)\right|_{t=1}$

(c) $\left.\dfrac{d}{dt}\left(f(t)f(t+2)\right)\right|_{t=3}$

(d) $\left.\dfrac{d}{dt}\left(\dfrac{f(t)}{g(t)}\right)\right|_{t=3}$

(e) $\left.\dfrac{d}{dt}\left(\dfrac{t^2+t}{f(t)}\right)\right|_{t=1}$

(f) $\left.\dfrac{d}{dv}\left(\dfrac{2^v - vg(v)}{f(1)g(v)+g(3)}\right)\right|_{v=5}$

(g) $g'(f(3))$

(h) $h'(3)$ if $h(p) = pf(p)g(p)$

2. Consider the piecewise-defined function

$$g(w) = \begin{cases} 2e^w + 5 & \text{if } w < 0 \\ 2w^2 + Aw + B & \text{if } w \geq 0 \end{cases}$$

where A and B are unknown constants.

Note: To gain insight into this problem, it may be useful to consider the graph of the function g for different values of A and B.
For each of the following, justify your answer.

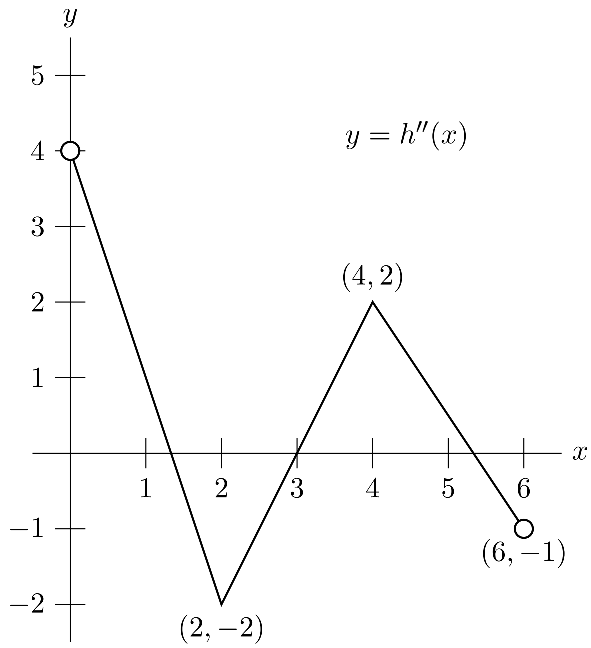
(a) Find values for A and B so that $g(w)$ is continuous and differentiable at $w = 0$. If no such values exist, explain why.

(b) Find values for A and B so that $g(w)$ is continuous but NOT differentiable at $w = 0$. If no such values exist, explain why.

(c) Find values for A and B so that $g(w)$ is differentiable but NOT continuous at $w = 0$. If no such values exist, explain why.

(d) Find values for A and B so that $g(w)$ is differentiable, and $g'(w)$ is continuous and differentiable at $w = 0$. If no such values exist, explain why.

3. Let $h(x)$ be a twice differentiable[1] function defined on the interval $0 < x < 6$. The graph of $y = h''(x)$ is shown below.



(a) When (i.e. over what intervals) is $h(x)$ concave up? When is $h(x)$ concave down?

(b) When is $h'(x)$ concave up? When is $h'(x)$ concave down?

(c) When is $h'(x)$ increasing? When is $h'(x)$ decreasing?

A new function $j(x)$ is defined by the equation

$$j(x) = 2h(x) - x^2 + 5x.$$

(d) When is $j(x)$ concave up? When is $j(x)$ concave down?

4. Let $r(x) = 4e^{x-2} - 2x + 3$. [Recall that $e^{x-2} = \frac{1}{e^2}(e^x)$.]

(a) Find a formula for the tangent line to the graph of $y = r(x)$ at $x = 2$.

(b) Find all values $c \neq 2$ (if any exist) such that the tangent line to the graph of $y = r(x)$ at $x = c$ is **parallel** to the tangent line to the graph at $x = 2$. Justify your answer.

(c) Find all values c (if any exist) such that the tangent line to the graph of $y = r(x)$ at $x = c$ is **perpendicular** to the tangent line to the graph at $x = 2$. Justify your answer.

[1] We say that a function is *twice differentiable* if it is differentiable and its derivative is also differentiable.