Math 115 - Team Homework Assignment #6, Winter 2016

- **Due Date:** March 15 or 16 (Your instructor will tell you the exact date and time.)
- Note: All problem, section, and page references are to the course textbook, which is the 6th edition of *Calculus: Single Variable* by Hughes-Hallett, Gleason, McCallum, et al.
- Remember to follow the guidelines from the "Doing Team Homework" and "Team HW Tutorial" links in the sidebar of the course website.
- Do not forget to rotate roles and include a reporter's page each week.
- Show ALL your work.

Note: Before beginning this assignment, be sure that you have carefully read the "Example of appropriate justification" document available on the Assignments page of the course website.

1. Let $h(t)$ be a twice differentiable function[1]. A table containing some values for $h'(t)$ and $h''(t)$ is given below.

t	0	1	2	4	5	6	8
$h'(t)$	3	0	1	2	0	-1	-7
$h''(t)$	-4	0	5	0	-3	0	-5

Assume that $h'(t)$ and $h''(t)$ are continuous and are always strictly increasing or always strictly decreasing between consecutive values t shown in the table. Assume additionally that $h(2) = 11$.

(a) Find all critical points of $h(t)$ for $0 < t < 8$ and classify each as a local minimum of h, a local maximum of h, or neither.

(b) Find all critical points of $h'(t)$ for $0 < t < 8$ and classify each as a local minimum of h', a local maximum of h', or neither.

(c) Find all inflection points of $h(t)$ for $0 < t < 8$.

(d) Let $L(t)$ be the local linearization of $h(t)$ at $t = 2$. Find a formula for $L(t)$.

(e) Using $L(t)$, approximate $h(2.1)$. Similarly, approximate $h(1.99)$. Are your estimates overestimates or underestimates? Explain.

Recall that for a function $f(t)$ such that f and f' are differentiable at $t = a$, we define "the quadratic approximation" [2] of $f(t)$ at $t = a$ to be the polynomial $Q(t)$ of degree (at most) 2 such that all of the following hold:

- $Q(t)$ and $f(t)$ have the same function value at $t = a$.
- $Q(t)$ and $f(t)$ have the same first derivative at $t = a$.
- $Q(t)$ and $f(t)$ have the same second derivative at $t = a$.

A formula for $Q(t)$ is given by

$$Q(t) = f(a) + f'(a)(t-a) + \frac{f''(a)}{2}(t-a)^2.$$

(f) Let $P(t)$ be the quadratic approximation of $h(t)$ at $t = 2$. Find a formula for $P(t)$.

(g) Using $P(t)$, approximate $h(2.1)$ and $h(1.99)$.

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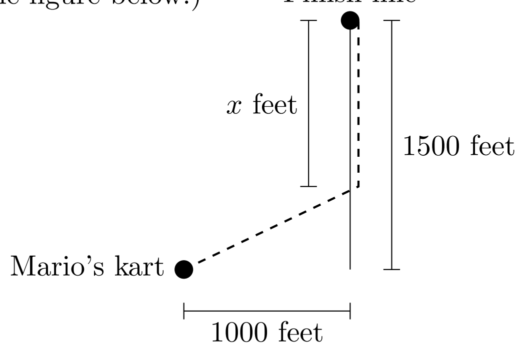
[1] By "twice differentiable", we mean that h is differentiable, and that its derivative h' is also differentiable.

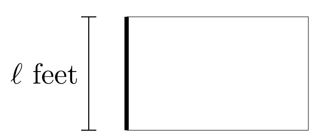
[2] $Q(t)$ is the Taylor polynomial of degree 2 for $f(t)$ at $t = a$

2. Mario is driving his kart on a straight road towards the finish line of a race. However, due to a series of mishaps, Mario suddenly finds himself on a grassy field 1000 feet west of the road. The finish line is 1000 feet east and 1500 feet north of Mario. The kart travels 30 feet/second on the grass, and 90 feet/second on the road. Mario will drive straight across the grass until he reaches a point on the road somewhere along the final 1500 foot stretch. Let $T(x)$ be the time (in seconds) it takes for Mario to reach the finish line if he drives straight across the grass until he reaches the point on the road that is x feet south of the finish line and then drives due north on the road for the remaining x feet. (See the figure below.)



   (a) Find a formula for $T(x)$.
   (b) In the context of this problem, what is the domain of $T(x)$?
   (c) What is the fastest time in which Mario can reach the finish line? Use calculus to find and justify your answer, and be sure to show enough evidence that you have indeed found the fastest time.

3. Gertrude wants to enclose a rectangular region in her backyard (as shown in the figure below). She wants to use extra tall fencing along one side (indicated by the bold line in the diagram), which costs \$150 per linear foot. For the remaining three sides, she wants to use normal fencing (thin line), which costs \$50 per linear foot. She plans to spend a total of \$2000 on the fencing for the project.



   (a) Let $A(\ell)$ be the area enclosed by the fence if Gertrude spends \$2000, where ℓ is the length (in feet) of the side with extra tall fencing. Find a formula $A(\ell)$.
   (b) In the context of this problem, what is the domain of $A(\ell)$?
   (c) What is the maximum possible area that Gertrude can enclose? Use calculus to find and justify your answer, and be sure to show enough evidence that you have indeed found the maximum possible area.

4. Let $g(x) = 3x^5 - 20x^3 + 100$.

   For each of the following, use calculus to find and justify your answers, and be sure to show enough evidence that you have found all of the points/values.

   (a) Find all critical points of $g(x)$.
   (b) Find and classify all local extrema of $g(x)$.
   (c) Find the global extrema (if they exist) of $g(x)$ on the interval $-3 \leq x \leq 1$.
   (d) Find the global extrema (if they exist) of $g(x)$ on the interval $-4 < x < 2.5$.
   (e) Find the global extrema (if they exist) of $g(x)$ on the interval $(-2.5, \infty)$.
   (f) Find all inflection points of $g(x)$.