Math 115 - Team Homework Assignment #7, Winter 2016

- **Due Date:** April 7 or 8 (Your instructor will tell you the exact date and time.)
- Note: All problem, section, and page references are to the course textbook, which is the 6th edition of *Calculus: Single Variable* by Hughes-Hallett, Gleason, McCallum, et al.
- Remember to follow the guidelines from the "Doing Team Homework" and "Team HW Tutorial" links in the sidebar of the course website.
- Do not forget to rotate roles and include a reporter's page each week.
- Show ALL your work.

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1. Bob is making artisanal soda to be sold to a retailer. The retailer will pay \$3 each for the first 1000 bottles, \$2 each for the next 2000 bottles (the 1001st bottle up to the 3000th bottle), and \$1 each for the remaining bottles (the 3001st bottle onwards).

   (a) Suppose Bob's cost, in dollars, for producing q bottles is given by $C(q) = \frac{1}{2000}q^2 + 1000$. Assuming that all bottles produced are sold to the retailer, how many bottles should Bob produce in order to maximize his profit? What is Bob's maximum possible profit? *Show all of your work, and use calculus to find and justify your solution.*

   (b) Suppose instead that Bob's cost for producing q bottles is given by $B(q) = \frac{1}{6000}q^2 + \frac{1}{2}q + 1000$. Assuming that all bottles produced are sold to the retailer, how many bottles should Bob produce in order to maximize profit? What is Bob's maximum possible profit? *Show all of your work, and use calculus to find and justify your solution.*

2. Let $v(t)$ be the velocity (in meters/second) of Alice's race car t seconds after the start of a race. Several measurements of $v(t)$ are given in the chart below.

t	0	10	25	30	50
$v(t)$	0	40	50	45	55

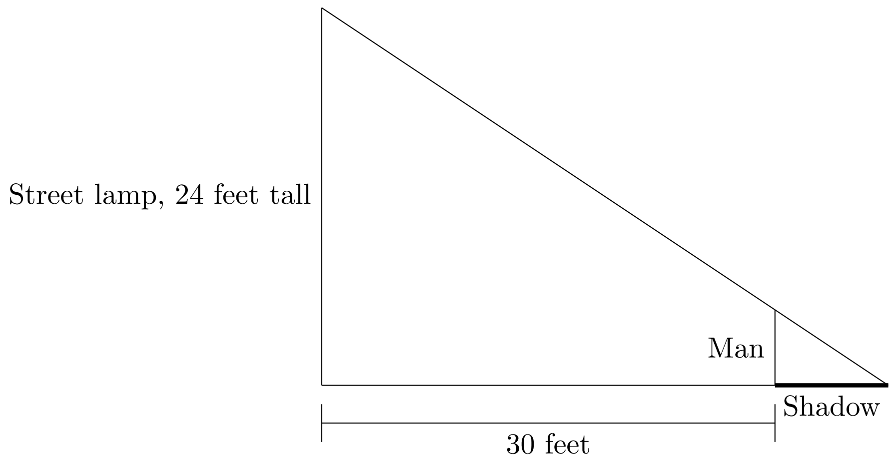
   Assume that between each pair of consecutive values of t shown in the table, $v(t)$ is either always strictly increasing or always strictly decreasing.

   (a) Based on the data provided, find the best possible upper estimate of the distance Alice's race car traveled during the first 50 seconds of the race.

   (b) Based on the data provided, find the best possible lower estimate of the distance Alice's race car traveled during the first 50 seconds of the race.

   (c) Suppose we wanted to estimate the distance Alice's race car traveled between 30 and 50 seconds after the start of the race to within 0.1 meters of the actual distance. How often must we take velocity measurements in order to be sure that we can determine this information?

3. A man, who is 30 feet away from a 24 foot tall street lamp, is sinking into quicksand. (*See diagram below.*) At the moment when 4 feet of him is above the ground, his height above the ground is shrinking at a rate of 0.5 feet/second.



(a) How long will the man's shadow (shown in bold in the diagram above) be at the moment when 4 feet of him is above the ground?

(b) At what rate is the length of the man's shadow changing at the moment 4 feet of him is above the ground? Is his shadow growing or shrinking at that moment?

4. A child psychologist is measuring a child's happiness after she lets go of a balloon. After letting the balloon go, the child becomes less happy as she faces the realization that the balloon is never coming back. Eventually, as the child forgets about the balloon, her happiness begins to return to its original levels.

The psychologist uses a unit of measurement called "unicorns" (where more unicorns means more happiness) to measure the child's happiness. The psychologist determines that an appropriate model for the the child's happiness (in unicorns) t minutes after she lets go of the balloon is

$$h(t) = ate^{bt} + c$$

for $t \geq 0$, for some constants a, b and c.

(a) If 0 unicorns is the lowest possible level of happiness, what do we know about the signs of the constants a, b, and c in the context of this problem?

(b) Suppose that the child's happiness reaches a minimum of 4 unicorns 2 minutes after she lets go of the balloon. After a long time, the child's happiness approaches 8 unicorns. Find values of the constants a, b, and c so that $h(t)$ models this behavior appropriately. (Once you find values of the constants, be sure to verify that your function does indeed have these properties.)