Math 115 — Final Exam

December 17, 2015

EXAM SOLUTIONS

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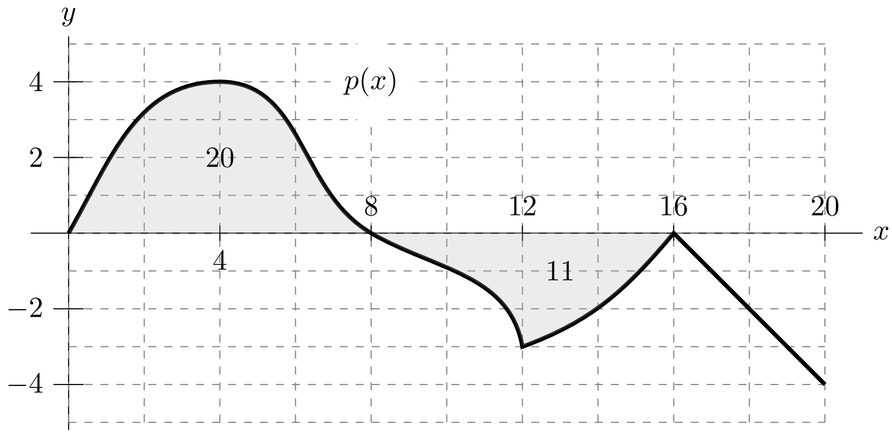
1. **Do not open this exam until you are told to do so.**
2. This exam has 11 pages including this cover. There are 11 problems.
   Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
3. Do not separate the pages of this exam. If they do become separated, write your initials (not name) on every page and point this out to your instructor when you hand in the exam.
4. Note that the back of every page of the exam is blank, and, if needed, you may use this space for scratchwork. Clearly identify any of this work that you would like to have graded.
5. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer questions about exam problems during the exam.
6. Show an appropriate amount of work (including appropriate explanation) for each problem, so that graders can see not only your answer but how you obtained it.
7. The use of any networked device while working on this exam is not permitted.
8. You may use any calculator that does not have an internet or data connection except a TI-92 (or other calculator with a "qwerty" keypad). However, you must show work for any calculation which we have learned how to do in this course. You are also allowed two sides of a $3'' \times 5''$ note card.
9. For any graph or table that you use to find an answer, be sure to sketch the graph or write out the entries of the table. In either case, include an explanation of how you used the graph or table to find the answer.
10. Include units in your answer where that is appropriate.
11. **Turn off all cell phones, smartphones, and other electronic devices**, and remove all headphones and smartwatches.
12. You must use the methods learned in this course to solve all problems.

Problem	Points	Score
1	12	
2	10	
3	12	
4	7	
5	9	
6	5	

Problem	Points	Score
7	6	
8	8	
9	10	
10	8	
11	8	
Post Test	5	
Total	100	

1. [12 points] Recall that a function h is odd if $h(-x) = -h(x)$ for all x. A portion of the graph of $p(x)$, an odd function, is shown below. Assume that the areas of the two shaded regions are 20 and 11, as indicated on the graph, and note that $p(x)$ is linear for $16 < x < 20$.



Remember to show your work throughout this problem.

a. [4 points] Compute the exact value of $\int_0^{20} (5 - 3p(x))\,dx$.

Solution: We have
$$\int_0^{20} (5 - 3p(x))\,dx = \int_0^{20} 5\,dx - 3\int_0^{20} p(x)\,dx$$
$$= 100 - 3(20 - 11 - \frac{4 \cdot 4}{2}) = 97.$$

Answer: 97

b. [2 points] Compute the exact value of $\int_4^8 p'(x)\,dx$.

Solution: By the Fundamental Theorem, we have $\int_4^8 p'(x)\,dx = p(8) - p(4) = 0 - 4 = -4$.

Answer: -4

c. [3 points] Find the average value of $p(x)$ on the interval $-16 \le x \le 8$.

Solution: The average value is given by $\frac{1}{8-(-16)}\int_{-16}^{8} p(x)\,dx$. Since p is odd, we have $\int_{-8}^{8} p(x)\,dx = 0$, and $\int_{-16}^{-8} p(x)\,dx = -\int_8^{16} p(x)\,dx$. Thus, the average value is
$$\frac{1}{24}\left(\int_{-16}^{8} p(x)\,dx\right) = \frac{1}{24}\left(\int_{-16}^{-8} p(x)\,dx + \int_{-8}^{8} p(x)\,dx\right) = \frac{1}{24}\left(-\int_8^{16} p(x)\,dx + 0\right) = \frac{11}{24}.$$

Answer: $\frac{11}{24}$

d. [3 points] Use a <u>right</u> Riemann sum with 3 equal subintervals to estimate $\int_{12}^{18} p(x)\,dx$. Write out all terms of the sum.

Solution:
$$2\,(p(14) + p(16) + p(18)) = 2(-2 + 0 + (-2)) = -8.$$

Answer: $\int_{12}^{18} p(x)\,dx \approx$ -8

2. [10 points] Let a be a constant with $a > 1$.
A function $w(x)$ and its derivative $w'(x)$ are given below.

$$w(x) = a + \frac{x}{x^2 + a^2} \quad \text{and} \quad w'(x) = \frac{-(x-a)(x+a)}{(x^2+a^2)^2}.$$

a. [5 points] Find and classify the local extrema of $w(x)$. Use calculus to find and justify your answers, and be sure to show enough evidence to demonstrate that you have found all local extrema. For each answer blank, write NONE if appropriate.

Solution: By inspection of the formula for w', we can see that the critical points of $w(x)$ are at $x = -a$ and $x = a$. (Note that $x^2 + a^2$ is never equal to 0 because $a \neq 0$.) The following sign chart shows the sign of $w'(x)$ on the intervals $-\infty < x < -a$, $-a < x < a$, and $a < x < \infty$. (Note that since a is positive, we have $-a < a$.)

Interval	$-\infty < x < -a$	$-a < x < a$	$a < x < \infty$
sign of $w'(x)$	$\frac{(-)(-)(-)}{+} = -$	$\frac{(-)(-)(+)}{+} = +$	$\frac{(-)(+)(+)}{+} = -$

By the First Derivative Test, we therefore see that $w(x)$ has a local minimum at $x = -a$ and a local maximum at $x = a$.

Answer: Local min(s) at $x =$ $-a$

Answer: Local max(es) at $x =$ a

b. [5 points] Find the global extrema of $w(x)$ on the interval $[1, \infty)$. Use calculus to find and justify your answers, and be sure to show enough evidence to demonstrate that you have found the global extrema. For each answer blank, write NONE if appropriate.

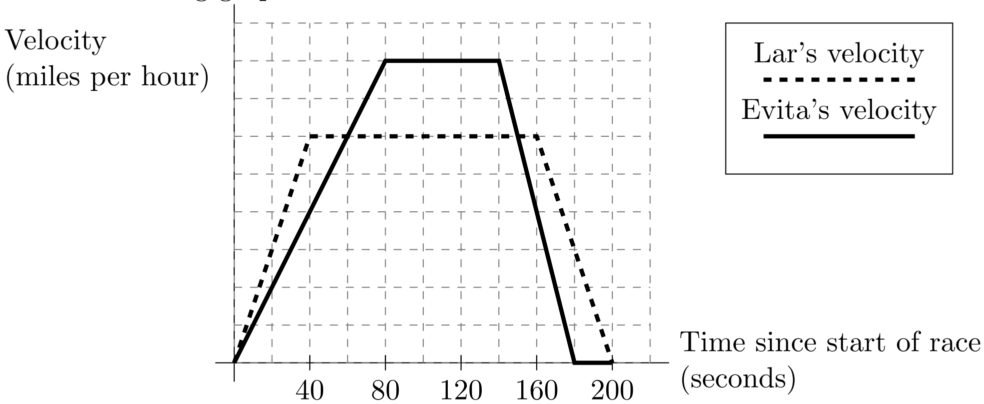
Solution:
One solution: In part (a) we showed that $x = a$ is a local max. Since $x = a$ is the only critical point in the interval $[1, \infty)$, we conclude that $x = a$ is the global max on this interval. To determine the global min, we need to consider what happens at $x = 1$ and as $x \to \infty$. We have $\lim\limits_{x \to \infty} w(x) = a$. We also have $w(1) = a + \frac{1}{1+a^2}$, which is larger than a, so $x = 1$ is not the global min. Since $w(x)$ decreases to a (but never quite reaches it) as $x \to \infty$, we conclude that $w(x)$ has no global min on the interval $[1, \infty)$.

Another solution: We check the values of $w(x)$ at $x = 1$ and $x = a$ (note that since $a > 1$, a is always in the interval $[1, \infty)$, and $-a$ is never in this interval) and then consider what happens as x goes to infinity.
We have $w(1) = a + \frac{1}{1+a^2}$ and $w(a) = a + \frac{1}{2a}$. Thus we need to compare $\frac{1}{1+a^2}$ and $\frac{1}{2a}$. These are equal if $a = 1$ (but we know $a > 1$), and otherwise, $\frac{1}{2a}$ is larger. Since $w(x)$ is decreasing for $x > a$, the global max is $\frac{1}{2a}$.
We have $\lim\limits_{x \to \infty} w(x) = a$ but $w(x)$ is never equal to a. Since this limit is smaller than $w(1)$ and $w(a)$, we conclude that $w(x)$ has no global minimum on the interval $[1, \infty)$.

Answer: Global min(s) at $x =$ NONE

Answer: Global max(es) at $x =$ a

3. [12 points] Lar Getni and Evita Vired run a <u>half-mile</u> race. After the race, C.T. Latnem Adnuf receives the following graph of the two runners' velocities over the course of the race.



Unfortunately, whoever made the graph forgot to label the scale of the vertical axis, and C.T. needs your help to answer the following questions. You may assume that the horizontal grid lines are evenly spaced, but do not assume that the scales of the two axes are the same. You may also assume that both runners completed the race and then stopped running.

a. [1 point] Who won the race?

Answer: **Evita**

b. [2 points] During what time interval(s) was Lar ahead of Evita?

Answer: $0 < t < 100$

c. [2 points] During what time interval(s) was Lar running faster than Evita?

Answer: $0 < t < 60$ **and** $150 < t < 200$

d. [4 points] What was the maximum speed (in miles per hour) attained by Lar? By Evita? *Remember to show your work.*

Solution: There are 48 boxes under the graph of Lar's (and also of Evita's) velocity. Let c denote the vertical dimension of a box, in miles per hour. The horizontal dimension of a box is 20 seconds, or $\frac{20}{3600}$ hours. Since the race is $\frac{1}{2}$ a mile long, and the area under the curve is equal to the distance traveled, we must have

$$48 \cdot c \cdot \frac{20}{3600} = \frac{1}{2}$$

so $c = 1.875$. Thus, the maximum speed attained by Lar is $6 \cdot 1.875 = 11.25$ mph, and the maximum speed attained by Evita is $8 \cdot 1.875 = 15$ mph.

Answer: Lar's max speed: **11.25 mph** and Evita's max speed: **15 mph**

e. [3 points] Let $v(t)$ (respectively, $w(t)$) be Evita's (respectively, Lar's) velocity in miles per hour t seconds after the start of the race. Write an equation involving one or more integrals that expresses the following statement:

N seconds after the start of the race, Evita is M miles ahead of Lar.

Your answer may involve $v(t)$ and $w(t)$.

Answer: $\dfrac{1}{3600}\displaystyle\int_0^N (v(t) - w(t))\,dt = M$

4. [7 points] Below is a table showing some values of an invertible and differentiable function m.

t	-0.11	-0.03	0	0.02	0.5	0.98	1	1.06	1.12
$m(t)$	1.548	1.423	1.000	0.721	0	-2.367	-2.441	-2.675	-2.913

Find the value of each of the quantities below. If it is not possible to find the value exactly, find the best estimate you can given all of the information provided above.

a. [1 point] $\lim_{t\to 0} m(t)$

Solution: Since m is differentiable, it is continuous, so $\lim_{t\to 0} m(t) = m(0) = 1.000$.

Answer: 1.000

b. [2 points] $(m^{-1})'(1)$

Solution: $(m^{-1})'(1) \approx \dfrac{m^{-1}(0.721) - m^{-1}(1)}{0.721 - 1} = \dfrac{0 - 0.02}{1 - 0.721} \approx -0.0717$

(We can also consider $\frac{m^{-1}(1.423) - m^{-1}(1)}{1.423 - 1} = \frac{-0.03 - 0}{1.423 - 1} \approx -0.0709$
or $\frac{m^{-1}(1.423) - m^{-1}(0.721)}{1.423 - 0.721} = \frac{-0.03 - 0.02}{1.423 - 0.721} = \frac{-0.03 - 0}{1.423 - 1} \approx -0.0712$.)

Answer: **Approximately** -0.07

c. [1 point] $\lim_{u\to 0} \dfrac{m(1+u) - m(1)}{u}$

Solution: $\lim_{u\to 0} \dfrac{m(1+u) - m(1)}{u} \approx \dfrac{m(1 + (-0.02)) - m(1)}{-0.02} = \dfrac{-2.367 - (-2.441)}{-0.02} = -3.7$

Answer: **Approximately** -3.7

Below is a table showing some values of another differentiable function n.
Assume that $n'(t)$ is continuous on the interval $[-0.1, 1.1]$.

t	-0.1	0	0.1	0.9	1	1.1
$n(t)$	2	-8	5	2	-2	3

Find the exact value of each of the quantities below. If it is not possible to find the value exactly, write NOT POSSIBLE.

d. [1 point] The average rate of change of $n(t)$ on the interval $[-0.1, 1.1]$

Solution: $\dfrac{n(1.1) - n(-0.1)}{1.1 - (-0.1)} = \dfrac{3 - 2}{1.2} = \dfrac{1}{1.2} = \dfrac{5}{6}$.

Answer: $\frac{5}{6}$

e. [1 point] $\int_0^1 n'(t)\,dt$

Solution: By the Fundamental Theorem of Calculus, we have

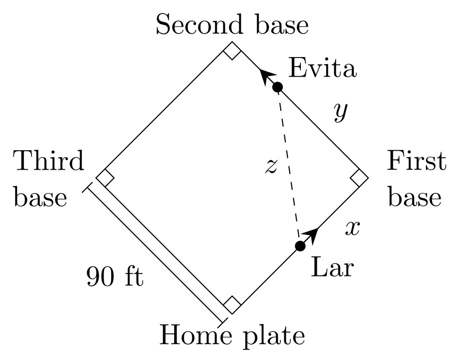
$$\int_0^1 n'(t)dt = n(1) - n(0) = -2 - (-8) = 6.$$

Answer: 6

f. [1 point] $\int_0^1 n(t)\,dt$

Answer: NOT POSSIBLE

5. [9 points]
During the annual Srebmun Foyoj kickball game, Lar Getni kicks the ball and runs from home plate to first base, while Evita Vired runs from first base to second base.



Let x be the distance between Lar and first base, y be the distance between Evita and first base, and z be the distance between Lar and Evita, as shown in the diagram on the right. Note that the bases are arranged in a square and that the distance between consecutive bases is 90 feet.

At the moment when Lar is halfway from home plate to first base, Evita is two thirds of the way from first base to second base. At this moment, Lar is running at a speed of 32 ft/s, and Evita is running at a speed of 36 ft/s. The questions below all refer to this moment.

Throughout this problem, remember to show your work clearly, and include units in your answers.

a. [5 points] At the moment when Lar is halfway to first base, at what rate is the distance between Lar and Evita changing? Is the distance increasing or decreasing?

Solution: We need to find $\frac{dz}{dt}$ at the moment when Lar is halfway to first base. By the Pythagorean Theorem, we have $z^2 = x^2 + y^2$. Differentiating both sides of this equation with respect to time, we find that $2z\frac{dz}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$.

At the moment in question, we have $x = 45$ ft, $y = 60$ ft, $z = \sqrt{45^2 + 60^2} = 75$ ft, $\frac{dx}{dt} = -32$ ft/s, and $\frac{dy}{dt} = 36$ ft/s. So at this moment,

$$\frac{dz}{dt} = \frac{2(45)(-32) + 2(60)(36)}{2(75)} = 9.6 \text{ ft/s}.$$

Answer: The distance is (circle one) $\boxed{\text{INCREASING}}$ DECREASING

at a rate of 9.6 **ft/s** .

b. [4 points] At the moment when Lar is halfway to first base, at what rate is the area of the right triangle formed by Lar, Evita, and first base changing? Is the area increasing or decreasing?

Solution: The area of the triangle is given by $A = \frac{xy}{2}$. Differentiating both sides of this equation with respect to time, we find that $\frac{dA}{dt} = \frac{x\frac{dy}{dt} + y\frac{dx}{dt}}{2}$.

At the moment when Lar is halfway to first base, we therefore have

$$\frac{dA}{dt} = \frac{(45)(36) + (60)(-32)}{2} = -150 \text{ ft}^2/\text{s}.$$

Answer: The area is (circle one) INCREASING $\boxed{\text{DECREASING}}$

at a rate of 150 $\mathbf{ft^2/s}$.

6. [5 points] Consider the differentiable function Z defined by

$$Z(v) = \begin{cases} \dfrac{e^{v-1} - v}{(v-1)^2} & \text{if } v \neq 1 \\ \frac{1}{2} & \text{if } v = 1. \end{cases}$$

Use the limit definition of the derivative to write an explicit expression for $Z'(1)$.
Your answer should not involve the letter Z. Do not attempt to evaluate or simplify the limit.
Please write your final answer in the answer box provided below.

Answer: $Z'(1) =$ $\boxed{\lim_{h\to 0} \dfrac{\frac{e^{1+h-1}-(1+h)}{(1+h-1)^2} - \frac{1}{2}}{h} \quad \text{or} \quad \lim_{h\to 0} \dfrac{\frac{e^{h}-1-h}{h^2} - \frac{1}{2}}{h}}$

7. [6 points] Consider the family of functions

$$g(x) = 16r^3 \ln(|x|) + \frac{1}{3}k^3x^3$$

where r and k are nonzero constants. Note that

$$g'(x) = \frac{1}{x}(k^3x^3 + 16r^3) \qquad \text{and} \qquad g''(x) = \frac{1}{x^2}(2k^3x^3 - 16r^3).$$

Find values of r and k so that $g(x)$ has an inflection point at $(1, 9)$. Be sure to justify that $(1, 9)$ is in fact an inflection point of $g(x)$ for your choice of r and k.

Solution: The candidates for inflection points are the values of x in the domain of $g(x)$ for which the second derivative is either zero or undefined. Since $x = 0$ is not in the domain of $g(x)$, the only candidate is when $2k^3x^3 - 16r^3 = 0$, or when $x = \dfrac{2r}{k}$.

So, in order for $g(x)$ to have an inflection point at $x = 1$, we must have $1 = \dfrac{2r}{k}$, or $k = 2r$.
In order for the point $(1, 9)$ to lie on the graph of $g(x)$, we need $g(1) = 9$. So we must have $g(1) = \frac{1}{3}k^3 = 9$, so $k = 3$, and $r = \dfrac{3}{2}$.

To justify that $(1, 9)$ is really an inflection point of $g(x)$, we will show that the second derivative changes sign across the point $x = 1$. If we plug in $k = 3$ and $r = \dfrac{3}{2}$ to $g''(x)$, then we get

$$g''(x) = \frac{1}{x^2}(54x^3 - 54) = \frac{54}{x^2}(x^3 - 1).$$

When $x > 1$, $(x^3 - 1)$ is negative and $\dfrac{54}{x^2}$ is positive, so $g''(x)$ is negative, and when $x < 1$, $g''(x)$ is positive because all terms are positive. Thus, $(1, 9)$ is indeed an inflection point.

Answer: $r =$ $\dfrac{3}{2}$ and $k =$ 3

8. [8 points] Elur Niahc keeps a bucket in his backyard. It contains water, and the water is two inches deep when a rainstorm starts. The storm lasts 20 minutes.

- Let h be the depth, in inches, of the water in the bucket.
- Let $V(h)$ be the volume, in gallons, of water in the bucket when the water is h inches deep. Assume that $V(h)$ is invertible and differentiable.
- Let $r(t)$ be the rate at which the volume of water in the bucket is increasing, measured in gallons per minute, t minutes after the storm starts. Assume that $r(t) > 0$ for the entire duration of the rainstorm.

For each of the questions below, circle the one best answer. No points will be given for ambiguous or multiple answers.

a. [2 points] Which of the following expressions represents the depth, in inches, of water in the bucket when the bucket contains 3 gallons of water?

i. $V(3)$ ii. $\boxed{V^{-1}(3)}$ iii. $2 + V(3)$ iv. $2 + V'(3)$

b. [2 points] Which of the following is the best interpretation of the equation $(V^{-1})'(3) = 0.4$?

i. The rate at which the depth of the water in the bucket is changing is increasing by 0.4 inches per minute when the bucket contains 3 gallons of water.

ii. During the third minute of the rainstorm, the volume of the water in the bucket increases by about 0.4 gallons.

iii. When the depth of the water in the bucket increases from 2.8 to 3 inches, the volume of the water increases by about 0.08 gallons.

iv. When the volume of the water in the bucket is 3 gallons, the depth of the water is about 0.2 inches less than the depth will be when the volume is 3.5 gallons.

c. [2 points] Which expression represents the volume, in gallons, of water in the bucket after the rainstorm ends?

i. $V\left(2 + \int_0^{20} r(t)\,dt\right)$

ii. $2 + \int_0^{20} r(t)\,dt$

iii. $\int_0^{20} V(2 + r'(t))\,dt$

iv. $\boxed{V(2) + \int_0^{20} r(t)\,dt}$

v. $2 + V(20)$

vi. $\int_0^{20} V'(t)\,dt$

d. [2 points] Which of the following represents the average rate of change of the volume, in gallons per minute, of the water in the bucket during the rainstorm?

i. $\dfrac{V(20) - V(0)}{20}$

ii. $\dfrac{r(20) - r(0)}{20}$

iii. $\boxed{\dfrac{1}{20}\int_0^{20} r(t)\,dt}$

iv. $\dfrac{1}{20}\int_0^{20} V(t)\,dt$

9. [10 points] Suppose $g(x)$ is a function and $G(x)$ is an antiderivative of $g(x)$ such that $G(x)$ is defined and continuous on the entire interval $-4 \le x \le 6$. Portions of the graphs of g and G are shown below. Note the following:

- $g(x)$ is zero for $-1 \le x \le 0$.
- For $4 \le x \le 6$, the graph of $g(x)$ is the lower half of the circle of radius 1 centered at $(5, 0)$.
- For $0 \le x \le 1$, the graph of $G(x)$ is the top right quarter of the circle of radius 1 centered at the origin.

a. [4 points] Use the portions of both graphs shown on the right to complete the table below with the exact values of $G(x)$.

x	-3	-1	4	6
$G(x)$	3	1	4	$4 - \frac{\pi}{2}$

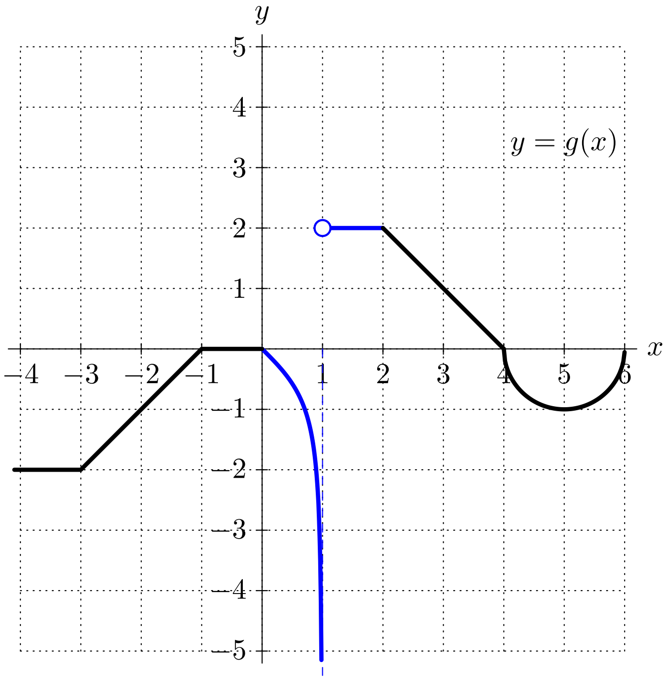
Solution: Note from the graph of G that $G(0) = 1$, $G(1) = 0$, and $G(2) = 2$. We find the values in the table by using the Fundamental Theorem of Calculus (and finding appropriate areas using the graph of $g(x)$).

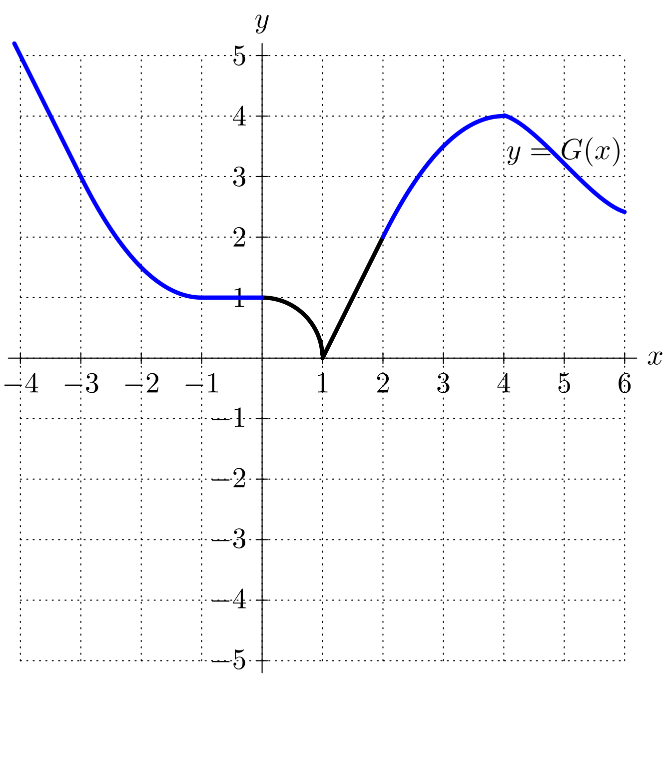
$$G(4) = \int_2^4 g(x)\,dx + G(2) = 2 + 2 = 4.$$

$$G(6) = \int_4^6 g(x)\,dx + G(4) = -\frac{\pi}{2} + 4.$$

$$G(-1) = G(0) - \int_{-1}^0 f(x)\,dx = 1 - 0 = 1.$$

$$G(-3) = G(-1) - \int_{-3}^{-1} f(x)\,dx = 1 - (-2) = 1.$$





b. [6 points] Use the axes on the right to sketch the missing portions of the graphs of g and G over the interval $-4 \le x \le 6$.
Be sure that you pay close attention to each of the following:

- the values of $G(x)$ you found in part (a) above
- where G is/is not differentiable
- where G and g are increasing, decreasing, or constant
- the concavity of the graph of $y = G(x)$

10. [8 points] Gen is setting up a business selling hot chocolate in Srebmun Foyoj and, due to local restrictions, she will be able to produce and sell no more than 200 gallons. She has determined that the total cost, in dollars, for her to produce g gallons of hot chocolate can be modeled by

$$C(g) = \begin{cases} 100 + 90\sqrt{g} & \text{if } 0 \le g \le 100 \\ 400 - 10e^5 + 6g + 10e^{0.05g} & \text{if } 100 < g \le 200 \end{cases}$$

and that for $0 \le g \le 200$, the revenue, in dollars, that she will bring in from selling g gallons of hot chocolate is given by

$$R(g) = 15g.$$

a. [4 points] For what quantities of hot chocolate sold would Gen's marginal revenue equal her marginal cost?

Solution: We have $R'(g) = 15$ and $C'(g) = \begin{cases} 45g^{-1/2} & \text{if } 0 \le g < 100 \\ 6 + 0.5e^{0.05g} & \text{if } 100 < g \le 200. \end{cases}$

For $0 \le g < 100$, marginal revenue is therefore equal to marginal cost when $45g^{-1/2} = 15$, so $g^{1/2} = 3$ and $g = 9$. When $100 < g \le 200$,

$$\begin{aligned} 6 + 0.5e^{0.05g} &= 15 \\ e^{0.05g} &= 18 \\ 0.05g &= \ln(18) \\ g = 20\ln(18) &\approx 57.8 \end{aligned}$$

However, this value of g is not in the domain of this piece, so MC and MR are never equal on this piece.

Note: We can also conclude that no such point exists on this interval by noting that since $6 + 0.5e^{0.05\cdot 100} > 80$ and MC is increasing, MC never equal 15.

Answer: 9 gallons

b. [4 points] Assuming Gen can sell up to 200 gallons of hot chocolate, how much hot chocolate should she produce in order to maximize her profit, and what would that maximum profit be? *You must use calculus to find and justify your answer. Be sure to provide enough evidence to justify your answer fully.*

Solution: Note that $C(g)$ is continuous, since $100 + 90\sqrt{100} = 1000$ and $400 - 10e^5 + 6 \cdot 100 + 10e^{0.05\cdot 100} = 1000$.

The profit function is given by $\pi(g) = R(g) - C(g)$. Since both $R(g)$ and $C(g)$ are continuous, we may, by the Extreme Value Theorem, consider only critical points and endpoints of the domain. The endpoints are at $g = 0$ and $g = 200$, and the critical points are at $g = 9$ (by previous part) and $g = 100$ (where MR is undefined).

$$\begin{aligned} \pi(0) &= 0 - 100 = -100 \\ \pi(9) = 15 \cdot 9 - (100 + 90\sqrt{9}) &= 135 - 370 = -235 \\ \pi(100) &= 1500 - 1000 = 500 \\ \pi(200) &\approx -217,380 \end{aligned}$$

(The last is because $\pi(200) = 15 \cdot 200 - (400 - 10e^5 + 6 \cdot 200 + 10e^{0.05\cdot 200})$
$= 3000 - (400 - 10e^5 + 1200 + 10e^{10}) \approx 3000 - 220380 = -217,380.$)

Therefore the max occurs at $g = 100$, which results in a profit of \$500.

Answer: gallons of hot chocolate: 100 and max profit: \$500

11. [8 points] You are not required to show your work on this page.

a. [2 points] A function $f(x)$ is differentiable. Some values of f and f' are shown in the table below.

x	0	1	2	3	4
$f(x)$	3	4	1	-1	-2
$f'(x)$	2	-2	-3	0	3

Let $g(x) = \cos(\frac{\pi}{2}f(x))$. Which of the following values of x <u>must</u> be a critical point of $g(x)$? Circle <u>all</u> such values.

0 $\boxed{1}$ 2 $\boxed{3}$ $\boxed{4}$ NONE OF THESE

b. [2 points] Which of the following expressions gives the linear approximation for $\arctan(x)$ near $x = 1$? Circle <u>all</u> such expressions.

i. $\boxed{\dfrac{\pi}{4} + \dfrac{1}{2}(x-1)}$

ii. $\dfrac{1}{2} + \dfrac{\pi}{4}(x-1)$

iii. $\dfrac{1}{1+x^2} + \dfrac{\pi}{4}(x-1)$

iv. $\arctan(x) + \dfrac{1}{2}(x-1)$

v. NONE OF THESE

c. [2 points] Which of the following functions are antiderivatives of $f(x) = \dfrac{1}{x}$? Circle <u>all</u> such functions.

i. $\ln(|x+1|)$

ii. $\boxed{\ln(|x|)}$

iii. $\boxed{\ln(|x|)+2}$

iv. $\boxed{\ln(4|x|)}$

v. $4\ln(|x|)$

vi. NONE OF THESE

d. [2 points] Suppose n is a positive integer, f is a decreasing, continuous function on the interval $[2,6]$, the value of the left Riemann sum with n equal subdivisions for $\int_2^6 f(x)dx$ is A, and $f(2) = f(6) + 8$. Circle <u>all</u> the statements that must be true.

i. $\boxed{A \text{ is an overestimate for } \int_2^6 f(x)dx.}$

ii. $\int_2^6 f(x)\,dx = 8$.

iii. $\boxed{\int_1^5 f(x+1)\,dx = \int_2^6 f(x)\,dx.}$

iv. The left Riemann sum for $\int_2^6 (f(x))^2\,dx$ with n equal subdivisions is equal to A^2.

v. NONE OF THESE