MATH 161 PRACTICE FINAL EXAM C

PART I (*6 pts each*) ***Answer any 17 of the following 21 questions.*** You need not justify your answer. You may answer more than 17 to obtain extra credit.

1. $\lim\limits_{n\to\infty} \dfrac{(2n+1)^3(n+2015)^5}{n(4n+3)(n-1492)^7}$

2. $\lim\limits_{x\to 0} \dfrac{\cos x - 1}{e^x - x^2 - x - 1}$

3. $\lim\limits_{x\to 0} \dfrac{\ln(ax+1)}{\ln(bx+1)}$ where *a* and *b* are positive constants.

4. $Let\ h(x) = \int_1^x \ln\left(1 + 2015 \ln t\right) dt$

   Compute $h''(\mathrm{e})$.

5. $\dfrac{d^{2015}}{dx^{2015}} \sinh(7x) =$

6. Solve the initial value problem:

$$\frac{dy}{dx} = \frac{\ln x}{x}$$ given that y = 2015 when x = 1.

7. Find an anti-derivative of $\dfrac{\left(1+\sqrt{x}\right)^{\frac{4}{5}}}{\sqrt{x}}$

8. Find an anti-derivative of:

$$\frac{1 + 3e^{3x} - e^{-x}}{e^{3x} + e^{-x} + x}$$

9. $\lim\limits_{x\to\infty} x\tan\left(\dfrac{2015}{x}\right) =$

10. Suppose that $\int_7^{13} f(x)\,dx = 3$ *and* $\int_7^{13} g(x)\,dx = 1$.

$Find\ \int_7^{13}\left(4f(x) - 3g(x) + 2\right)dx$

11. Find the *average value* of the function y = $\sec^2$ x over the interval [0, π/4].

(Give the precise result without rounding.)

12. Find the value of *c* such that the conclusion of the Mean Value Theorem is verified for the function $g(x) = \dfrac{1}{(x-1)^2}$ on the interval [2, 5]. Express your answer to the nearest hundredth.

13. $Find\ \lim\limits_{n\to\infty}\left(\sqrt{n^2 + 92n + 1789} - n\right)$

14. Let *a* and *b* be non-zero constants. $Let\ f(x) = \dfrac{x+a}{x^2+b}$.

Find the *slope of the tangent line* to y = f(x) at x = 0. (Your answer may include the constants *a* and *b*.)

15. Let a and b be non-zero constants. Then $\int \frac{\sec x \tan x}{a + b \sec x}\, dx =$

16. Suppose that $\int_1^x g(t)\, dt = x^3 - 1$. Find g(5).

17. *Let* $y = x^{x^2+x+1}$. Find dy/dx when x = 1.

18. *Compute* $\int_{-1}^{3} |x|\, dx.$

19. *Compute* $\lim_{n\to\infty} \sum_{j=1}^{n} \frac{1}{n} \cos\left(\frac{j}{n}\pi\right)$ (*Hint:* Convert this limit into a Riemann integral.)

20. *Given that* $G(x) = \int_0^{3x^2} \sqrt{1+t^3}\, dt$, *find* $G'(1)$.

21. Charlotte, the spider, lives on the x-axis. Suppose that at time t = 1 minute, she is at x = 5 cm, and that her velocity (in cm/minute) at time *t* is given by: $v(t) = 4t^3 - 6t^2 + 1$. *Where* is Charlotte at time *t = 2 minutes*?

PART II *(12 pts each)*

Answer any 11 of the following 14 problems. You may answer more than 11 for extra credit.

1. Find the equation of the tangent line to the curve defined implicitly by

$$x^4 + y^3 - x^2 y = 13 + \ln y$$

at the point P = (2, 1).

2. Gilberte, who is 5 feet tall, walks away from an 18 foot lamppost. She observes that when she is 8 feet from the base of the lamppost, her shadow is increasing at a rate of 6 ft/min. Find Gilberte's speed when she is 8 feet from the base of the lamppost.

3. Using an *appropriate tangent line approximation*, estimate the value of $\sqrt[5]{1.0004}$. Have you obtained an overestimate or an underestimate? Explain. *Sketch!*

4. Albertine wishes to approximate a root of $g(x) = x^4 + x - 1$. Note that $g(0) < 0$ and $g(1) > 0$.

   (a) How does Albertine know that there must exist a solution to $g(x) = 0$ in the interval (0, 1)?

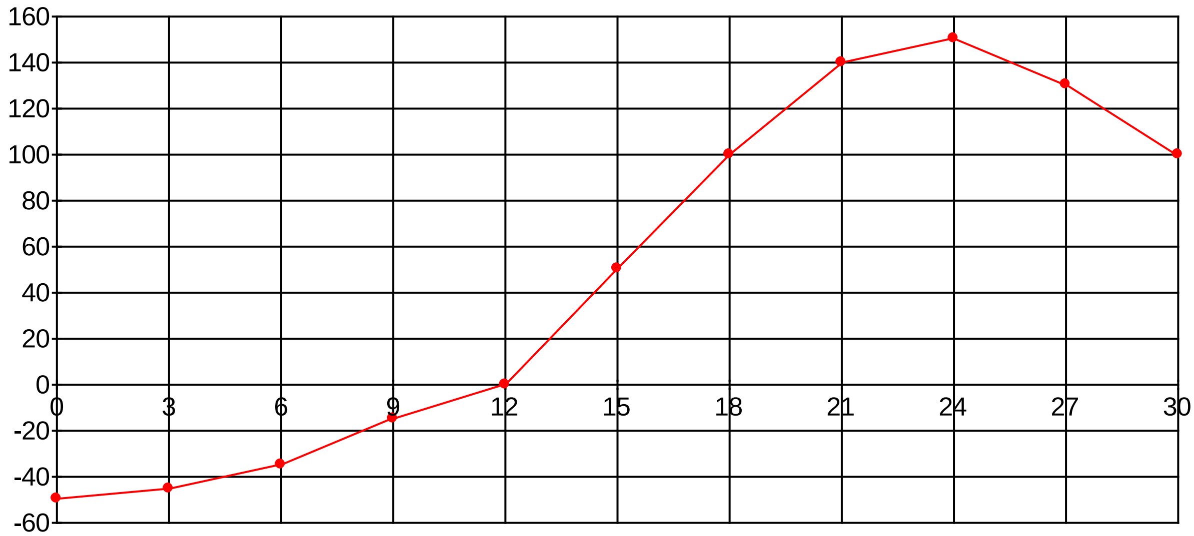
   (b) Let Albertine's initial guess for the root be $x_1 = 0.5$. Using Newton's method, which values would she obtain for x_2 and x_3? (Express your answers to the nearest thousandth.)

5. Graph the function $f(x) = (x-1)^2 e^x$. Identify any and all local and global extrema and points of inflection.

6. Madam Verdurin is building an open planter in the shape of a rectangular box with a square base. The base is made of metal that costs $7 per square foot. The sides are made of wood that costs $3 per square foot. The planter must hold at least 8 cubic feet of dirt. Find the dimensions of the *least expensive* planter that Madame Verdurin can build.

7. The graph below shows the *RATE OF CHANGE* of the quantity of water in the Water Tower of OZ, in liters per day, during the month of April, 2015. The tower contained 12,000 liters of water on April 1. *Estimate* the quantity of water in the tower on April 30. Show your work.

Rate of Change of Quantity of Water



8. Using the FTC, find the area bounded by the two parabolas:

$y = x^2 - 5x$ and $y = 20 + x - x^2$. *Sketch.*

9. Use a *left-endpoint* Riemann sum with $n = 4$ rectangles to approximate the area under the curve $f(x) = \dfrac{1}{x^3 + 1}$ between x = 0 and x = 2. Draw a picture to illustrate what you are computing. Is this an *underestimate* or an *overestimate* of the area? Explain!

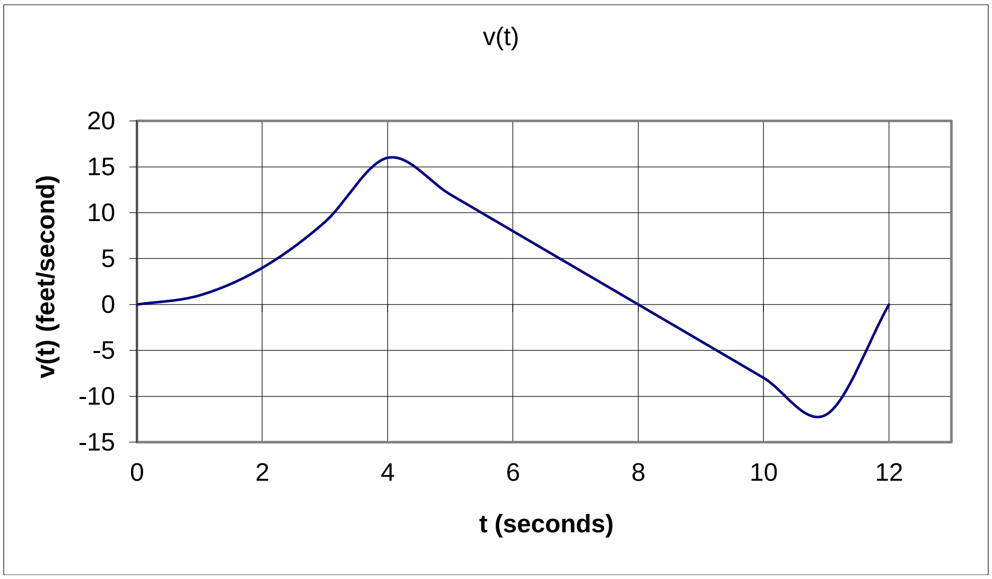
11. Graph the cubic polynomial $g(x) = x^3 + x^2 - 8x + 5$. Identify any and all local and global extrema and points of inflection.

12. The function y = F(x) is defined below:

$$F(x)=\begin{cases}\dfrac{3x^4-2x^3-21x^2}{x-3} & if\ \ x\neq 3\\ \\ k & if\ \ x=3\end{cases}$$

For which value(s) (if any) of *k* is the function *everywhere continuous*? Explain!

13. Albertine launches a model rocket from the ground at time t = 0. The rocket starts by traveling straight up in the air. The graph below illustrates the upward velocity of the rocket as a function of time.



(a) Sketch a graph of the *acceleration* of the rocket as a function of time.

(b) Sketch a graph of the *height* of the rocket as a function of time.

(c) Give an estimate of the *maximum height* the rocket achieved.

14. (University of Michigan)

[8 points] A ship is sailing out to sea from a dock, moving in a straight line perpendicular to the coast. At the same time, a person is running along the coast toward the dock, hoping desperately to jump aboard the departing ship. Let $b(t)$ denote the distance in feet between the ship and the dock t seconds after its departure, and let $p(t)$ denote the distance in feet between the person and the dock t seconds after the ship's departure. The situation is depicted below for your reference:

Suppose that 10 seconds after the ship's departure, it is 40 feet from the dock and is sailing away at a speed of 20 ft/sec. At the same moment, the person is 30 feet from the dock and running toward it at 14 ft/sec.

a. [2 points] What is $b'(10)$? What is $p'(10)$?

b. [6 points] Is the distance between the person and the ship increasing or decreasing 10 seconds after the ship's departure? How fast is it increasing or decreasing? (Include units in your answer, and keep in mind that distance is measured along a straight line joining the person and the ship.)

But in the new approach, as you know, the important thing is to understand what you're doing, rather than to get the right answer.

- Tom Lehrer (American singer-songwriter, satirist, pianist, and mathematician.)