# PRACTICE PROBLEMS FOR QUIZ I (REVISED)

1. Label each of the following functions as "odd", "even" or "neither odd nor even." (You need not justify your answers.)

(a) $y = \ln(1 + x^4) + 1/(1 + x^2)$

(b) $y = x^3 + \sin(x^5) + x \cos x$

(c) $y = 1 + x^2 + x^3$

(d) $y = x^5 \cosh x$

(e) $y = (x + \sinh x)^2$

2. For which non-zero value of *k* will the following quadratic equation have *only one* real root?

$$7x^2 + kx + 3k = 0$$

3. Consider the following rational functions. For each function, determine the limiting behavior as $x \to \infty$. Briefly explain how you arrived at your answers.

(a) $y = \dfrac{(2x-5)^2}{x^2}$

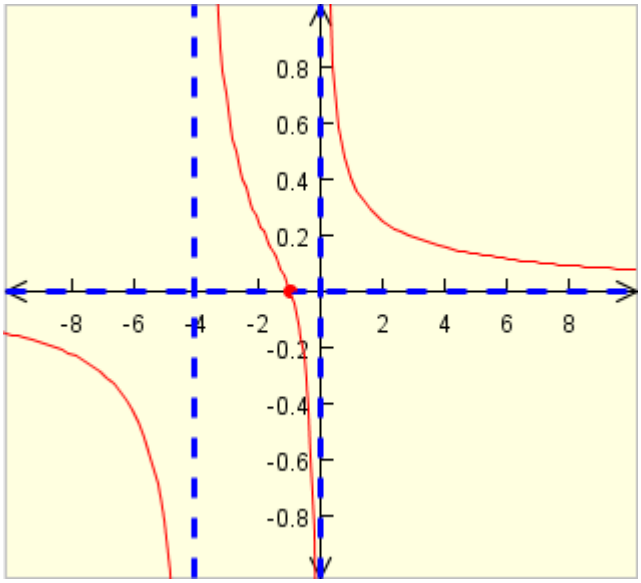
Answer: As $x \to \infty$, $y \to$ _____

(b) $y = \dfrac{(x+5)^2(x-11)^3}{(x-9)(x+13)(x-1)(7x-44)}$

Answer: As $x \to \infty$, $y \to$ _____

(c) $y = \dfrac{99(x-15)(3x+11)}{(x-9)^2(19x+13)}$

4. Find an *equation* of a rational function whose graph is given below:




5. Sketch the graph of the following function (that includes all the significant properties):

$$f(x) = \frac{(x-3)^2(x-4)^3(x-5)^5}{x^2(2x^2+x+1)^4}$$

6. Using an appropriate table (for example, letting x = 0.1, 0.01, 0.001, etc.) determine (to the nearest tenth) the behavior of the function

$f(x) = (1+3x)^{\frac{2}{x}}$ as x approaches 0 from the right. (*Show your work!*)

| *x* | *f(x)* |
|---|---|
| 1 | |
| 0.1 | |
| 0.01 | |
| 0.001 | |
| 0.0001 | |
| 0.00001 | |
| 0.000005 | |

7. Compute each of the following limits. (Explain your reasoning. You may use estimation techniques, tables, graphing calculators, etc.)

(a) $\lim_{x\to\infty} \dfrac{(x+11)^2(3x-7)^3}{\left(2x^2+4\right)^4(x+2015)}$

(b) $\lim_{x\to\infty} \dfrac{1+\sqrt{x}}{5+x^2}$

(c) $\lim_{x\to\infty} \dfrac{\sinh x+\cosh x}{e^x}$

(d) $\lim_{x\to 0} x^4\cos\left(\dfrac{1}{x}\right)$

(e) $\lim_{x\to 2} \dfrac{\dfrac{1}{x}-\dfrac{2}{x^2}}{2-x}$

(f) $\lim_{x\to 0}\left(e^{x^2}-x^{e^2}\right)$

(g) $\lim_{x\to 3/2}\dfrac{24x^3+7x^2-15x}{4x^3+4x^2-15x}$

8. *Estimate* the following limit by substituting appropriate values of *x*. (Show your results in tabular form. Give your answer to two decimal places.)

$$\lim_{x\to 1}\frac{5^x-5}{x-1}$$

9. Use the graph below to find approximate values for each of the following limits (if they exist).

(*a*) $\lim_{x\to -4} f(x)$

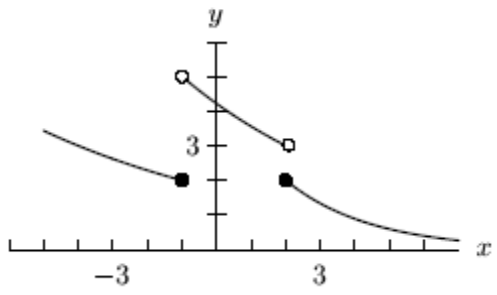
(*b*) $\lim_{x\to -1} f(x)$

(*c*) $\lim_{x\to 2} f(x)$

(*d*) $\lim_{x\to 6} f(x)$

(*e*) $\lim_{x\to -1+} f(x)$

(*f*) $\lim_{x\to 2-} f(x)$




10. Calculate each of the following limits or explain why the limit does not exist. Justify each answer. If you use the *Squeeze Theorem*, be precise.

(a) $\lim_{x\to 3} \dfrac{x^4 - 11}{(x-3)^4}$

(b) $\lim_{x\to 0} x^8 \cos^4(1/x)$

(c) $\lim_{x\to \infty} \dfrac{\sin(\ln(5+x))}{x+1}$

(d) $\lim_{x\to 2} \dfrac{x^4 - 16}{x-2}$

11. Compute each of the following limits or explain why the limit fails to exist. Justify your reasoning. (A calculator solution earns only partial credit.)

(a) $\lim_{x\to 2-} \dfrac{x^2(x-2)(x+3)}{|x-2|}$

(b) $\lim_{x\to 1}\left(\dfrac{\frac{1}{x^2}-1}{x-1}\right)$

(c) $\lim_{x\to 0}\dfrac{x}{\cos 9x}$

(d) $\lim_{x\to 0}\sin\dfrac{1}{x}$

(e) $\lim_{x\to\infty}\left(\dfrac{\sin x}{x}+\cos\left(\dfrac{13}{x}\right)\right)$

12. Consider the rational function F defined by $F(x)=\dfrac{15x^3+x^2-6x}{6x^2+x-2}$

(a) Where is *F undefined*? (*Hint:* Your answer should consist of two *x* values.)

(b) Let *p* denote the smaller of the two numbers found in part (a). Is it possible to *extend F* to a function that is continuous at x = p? Explain.

(c) Let *q* denote the larger of the two numbers found in part (a). Is it possible to extend *F* to a function that is continuous at x = q? Explain.

13. Compute (showing your work):

$$\lim_{x\to 0+}\left(\tanh\left(\frac{1}{x}\right)+e^{-\frac{1}{x}}+\frac{\sin\left(\frac{5}{x}\right)}{\ln x}\right)$$

14. Does the following limit exist? Explain why or why not.

$$\lim_{x\to 13}\frac{|3x-39|}{x-13}$$

15. Calculate each of the following limits. Briefly justify each answer.

(*a*) $\lim_{x\to 1}\frac{x^8-1}{x^3-1}$

(*b*) $\lim_{x\to 0} x^4\cos(1/x)$

(*c*) $\lim_{x\to\infty} x\sin(1/x)$

16. Compute each of the following one-sided limits or explain why the limit doesn't exist.

(*a*) $\lim_{x\to 1+}\frac{x-1}{|x-1|}$

(*b*) $\lim_{x\to 1-}\frac{x-1}{|x-1|}$

18. Let y = g(x) be defined as follows

$$g(x)=\begin{cases}3-x \ \ when \ \ x<2\\ 2 \quad if \ \ x=2\\ \dfrac{x}{2} \ \ if \ \ x>2\end{cases}$$

(a) Sketch the curve.

(b) Compute each of the following or explain why the limit fails to exist.

$(i)\ \lim\limits_{x\to 2} g(x), \quad \lim\limits_{x\to 2-} g(x), and\ g(2)$

$(ii)\ \lim\limits_{x\to 2} g(x)$

$(iii)\ \lim\limits_{x\to -1-} g(x), \quad \lim\limits_{x\to -1+} g(x)$

$(iv)\ \lim\limits_{x\to -1} g(x)$

19. Which, if any, of the following functions possess a limit as x→0? Briefly explain.

(a) y = 1/x

(b) y = sin (1/x)

(c) y = ln x

(d) y = exp(1/x)

$(e)\quad y = \arctan\left(\frac{1}{x^2}\right)$

$(f)\quad \lim_{x\to 0} \sqrt{\frac{\sin\left(x+\frac{\pi}{2}\right)}{(3x+1)^{75}}}$

20. Does there exist a *continuous extension* to the curve

$$g(x) = \frac{3x^2 - 4x + 1}{x^4 - 1}$$

at x = 1? If so, find it; if not explain!

21. Classify the *type of discontinuity* for each of the following:

$(a)\quad \frac{|x|}{x} \quad at\ x = 0$

$(b)\quad \cos\frac{1}{x} \quad at\ x = 0$

$(c)\quad \frac{1}{(x-3)^4} \quad at\ x = 3$

$(d)\quad \frac{2x^2 - 9x - 35}{x - 7} \quad at\ x = 7$

$(e)\quad \frac{12x^2 - 7x - 10}{x - 5} \quad at\ x = 5$

22. Using an appropriate table, determine the behavior of the function $f(x) = x^{\ln x}$ as x approaches 0 from the right.

*To be pleased with one's limits is a wretched state.*

- **Johann Wolfgang von Goethe** (1749 - 1832)