MATH 161 PRACTICE QUIZ II

1. Carefully state the *Squeeze Theorem*. Using the Squeeze Theorem compute each of the following limits:

$(a)\ \lim_{x\to 0} x^8 \sin^4(1/x)$

$(b)\ \lim_{x\to 0} x^4 \cos(1/x)$

$(c)\ \lim_{x\to \infty} x\sin(1/x)$

$(d)\ \lim_{x\to \infty} \dfrac{x^2\cos(2x)+\sin^3(x^{2015})}{x^3+x+5}$

2. (a) State carefully the *Intermediate Value Theorem.*

(b) Using the Intermediate Value Theorem, explain why the polynomial function $g(x) = x^5 - 4x^3 + 3x - 1$ has at least one real positive root x.

3. Compute $\lim_{x\to 0} \dfrac{\sin 5x}{\tan 11x}$. Show your work.

4. Compute $\lim_{x\to 0} \dfrac{\sin(3\cos x)}{\cos(\sin x)}$. Show your work.

5. Carefully state the *Intermediate Value Theorem*. Let $f(x) = 7 + 2x - x^3$ be defined on the interval [1, 3].

   (a) Explain why *f* must assume the value 0 somewhere on this interval.

   (b) Must *f* assume the value -13 on the interval [1, 3]? Does the Theorem imply that *f* must assume the value 9.3 on the interval [1, 3]?

6. Compute $\lim_{x\to 0}\left(\frac{\tan^3 5x}{\tan^3 2x} + x\csc\frac{x}{2} + x\sin\frac{3}{x}\right)$. Show your work.

7. Compute $\lim_{x\to 0}\frac{\sin ax}{\sin bx}$. Have you made any assumptions about the constants *a* and *b*?

8. Charlotte the spider lives on the x-axis. Assume that Charlotte was born at time *t = 0* days and dies at time *t = 13* days. Her position at time *t* (days) is given by $x(t) = t^2(t-2)$ feet.

   (a) Find Charlotte's *position* at time t = 4 days.

   (b) When does Charlotte find herself to the *left of the origin*?

   (c) Find Charlotte's *average velocity* during her lifetime.

   (d) Find Charlotte's *average velocity* during the time interval $4 \le t \le 4 + h$. *Simplify* your answer.

9. (University of Michigan problem)

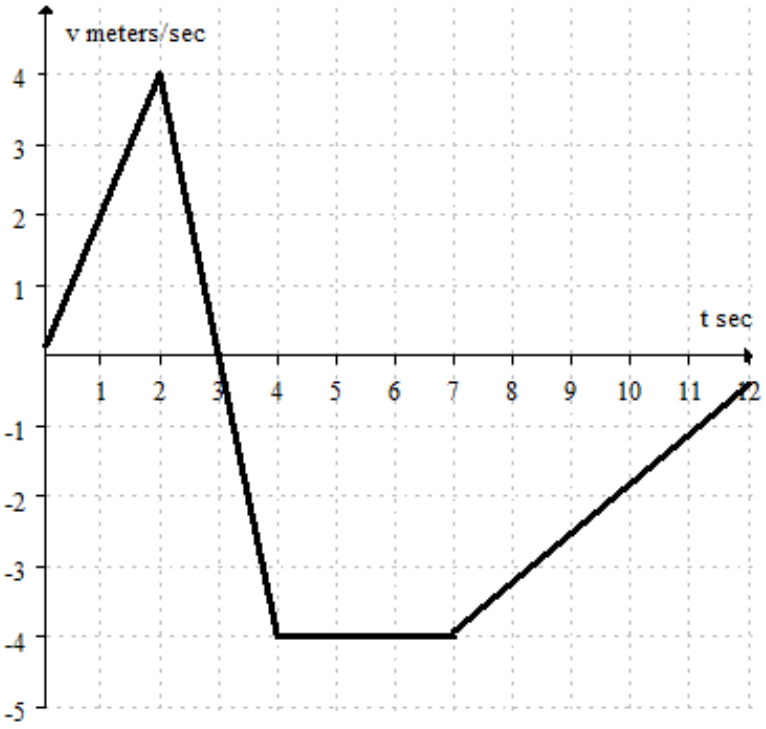
A runner competed in a half marathon in Anaheim, a distance of 13.1 miles. She ran the first 7 miles at a steady pace in 48 minutes, the second 3 miles at a steady pace in 28 minutes and the last 3.1 miles at a steady pace in 18 minutes.

a) Sketch a well-labeled graph of her distance completed with respect to time.

b) Sketch a well-labeled graph of her velocity with respect to time.

10. (University of Michigan problem)

The graph below shows the velocity of a bug traveling along a straight line on the classroom floor.



At what time(s) does the bug turn around?

A) At 3 seconds.
B) At 2 seconds and again at 7 seconds.
C) At 4 seconds and again at 7 seconds.
D) Never.

"Alice laughed: "There's no use trying," she said; "one can't believe impossible things."
"I daresay you haven't had much practice," said the Queen.
"When I was younger, I always did it forhalf an hour a day.
Why, sometimes I've believed as many as six impossible things before breakfast."

- Lewis Carroll, **Alice in Wonderland.**