MATH 161 PRACTICE QUIZ III

1. Using only the *definition* of derivative, find the derivative of the function

   $f(x) = ax^2 + bx + c$.

2. Using only the *definition* of derivative, find the derivative of the function

   $f(x) = x^4 + 1789$. (Here you may wish to use Pascal's triangle to shorten your algebraic work.)

3. The parabola $y = x^2 + x + c$ is tangent to the line $y = 3x$. Find c. (Include a picture in your explanation.)

4. Find a parabola, $y = ax^2 + bx + c$, that passes through the point (1, 4) and whose tangent lines at x = -1 and x = 5 have slopes 6 and -2 respectively.

5. Find equations of any (and all) tangent lines to the parabola $y = x^2 + 1$ that have *x-intercept* of -4/3. Sketch.

6. Archy lives on the x-axis. His position at time *t* (hours) is

   $s(t) = 4t^3 - 15t^2 + 12t + 1$ (cm).

   Assume that he was born at time t = 0.

   (a) What is Archy's *position* at time t = 1?

   (b) What is Archy's instantaneous *velocity* at time t = 1?

   (c) When is Archy moving *toward the left*? (Give one or more time intervals.)

7. The quantity, *Q* mg, of nicotine in the body *t* minutes after a cigarette is smoked is given by $Q = g(t)$.

   (a) Using a complete sentence, interpret the statement g(20) = 0.36.

   (b) Using a complete sentence, interpret the statement g′(20) = -0.002.

   (c) Using the information that you obtained above, estimate g(23).

8. Consider the function $f(x) = 2x^3 - 3x^2 - 12x + 2015$

Find any and all points (only their x-coordinates) at which the tangent line to y = f (x) is horizontal. (You may assume that $df/dt = 6x^2 - 6x - 12$)

9. The cost of extracting *T* tons of ore from a copper mine is $C = F(T)$ dollars.

(a) Using a complete sentence that avoids mathematical terminology, explain the meaning of $F(2000) = 300{,}000$. (Include appropriate units.)

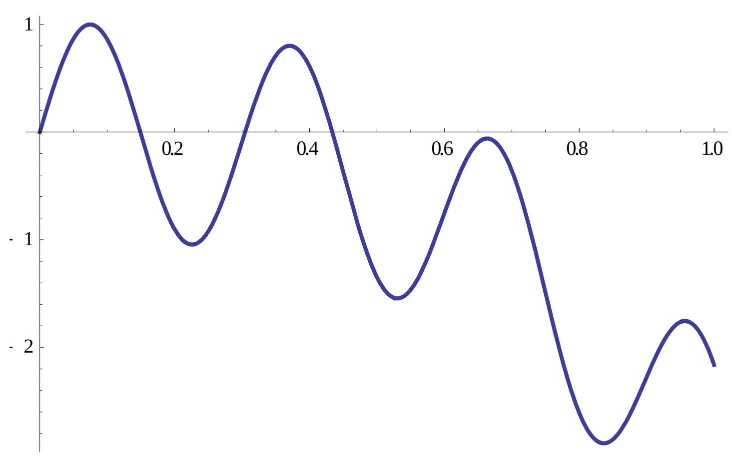
(b) Using a complete sentence that avoids mathematical terminology, explain the meaning of $F'(2000) = 131$. (Include appropriate units.

(c) Using the information above, estimate the cost of extracting 2,125 tons of ore from the mine.

10. Albertine travels from Chartres to Mt. Saint Michelle at an average speed of 50 km/hr. She returns to Chartres at an average speed of 60 km/hr. What is Albertine's *average speed* during the roundtrip?

11. Given the following graph of $y = f(x)$, use "geometric differentiation" to sketch the graph of dy/dx.

(If you wish to know, the equation of this curve is $y = x^5 + \sin(21x) - 4x^3$)



12. Given $f(x) = x^3 - 6x^2 + 9x - 5$.

(a) Find the slope of the tangent line to the curve at $x = -2$. (You may assume that $df/dx = 3x^2 - 12x + 9$.)

(b) What is the equation of this tangent line?

(c) What is the equation of the normal line at $x = -2$.

(d) Find all points where the curve has a horizontal tangent.

13. Using only the definition of derivative, find the derivative of the function $g(x) = \frac{1}{\sqrt{x+1}}$ at x = 3. Next, find the equation of the tangent line to this g(x) at x = 3.

14. Show why (d/dx) sin x = cos x.

15. Does the curve y = x^3 + x + 1 ever have a horizontal tangent line? If so, where?

16.

The expression

$$\frac{V(3) - V(1)}{3 - 1}$$

represents

(a) The average rate of change of the radius with respect to the volume when the radius changes from 1 inch to 3 inches.
(b) The average rate of change of the radius with respect to the volume when the volume changes from 1 cubic inch to 3 cubic inches.
(c) The average rate of change of the volume with respect to the radius when the radius changes from 1 inch to 3 inches.
(d) The average rate of change of the volume with respect to the radius when the volume changes from 1 cubic inch to 3 cubic inches.

17.

[12 points] A paperback book (definitely not a valuable calculus textbook, of course) is dropped from the top of Dennison hall (which is 40 m high) towards a very large, upward pointing fan. The average velocity of the book between time $t = 0$ and later times is shown in the table of data below (in which t is in seconds and the velocities are in m/s).

between $t = 0$ seconds and $t =$	1	2	3	4	5
average velocity is	-5	-10	-11.67	-9	-7.2

a. [8 points] Fill in the following table of values for the height $h(t)$ of the book (measured in meters). Show how you obtain your values.

t	0	1	2	3	4	5
$h(t)$	40	____	____	____	____	____

Which of the following represents the rate at which the volume is changing when the radius is 1 inch?

(a) $\frac{V(1.01) - V(1)}{0.01} = 12.69 \text{ in}^3$
(b) $\frac{V(0.99) - V(1)}{-0.01} = 12.44 \text{ in}^3$
(c) $\lim_{h \to 0} \left(\frac{V(1+h) - V(1)}{h} \right) \text{ in}^3$
(d) All of the above

18.

Which of the following expressions represents the slope of a line drawn between the two points marked in Figure 2.5?

(a) $\dfrac{F(\Delta x) - F(x)}{\Delta x}$ (b) $\dfrac{F(x+\Delta x) - F(x)}{\Delta x}$ (c) $\dfrac{F(x+\Delta x) - F(x)}{x}$

(d) $\dfrac{F(x+\Delta x) - F(x)}{x + x - \Delta x}$ (e) $\dfrac{F(x+\Delta x) - F(x)}{x + \Delta x}$

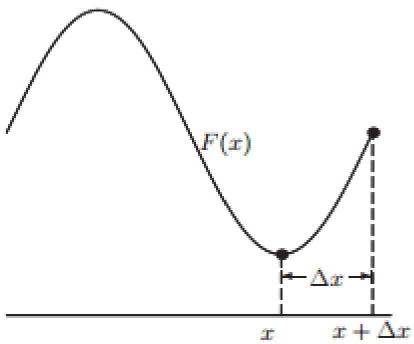


Figure 2.5

19.

Let $f(x) = x|x|$. Then $f(x)$ is differentiable at $x = 0$.

(a) True
(b) False

20.

Which of the following graphs (a)–(d) could represent the slope at every point of the function graphed in Figure 2.6?

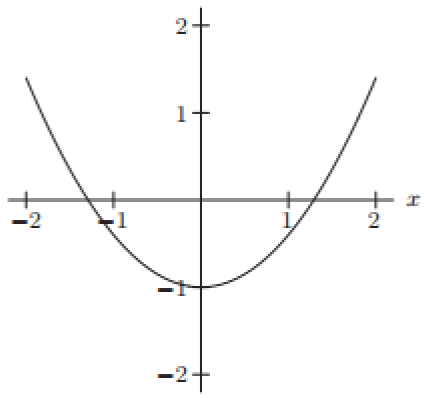
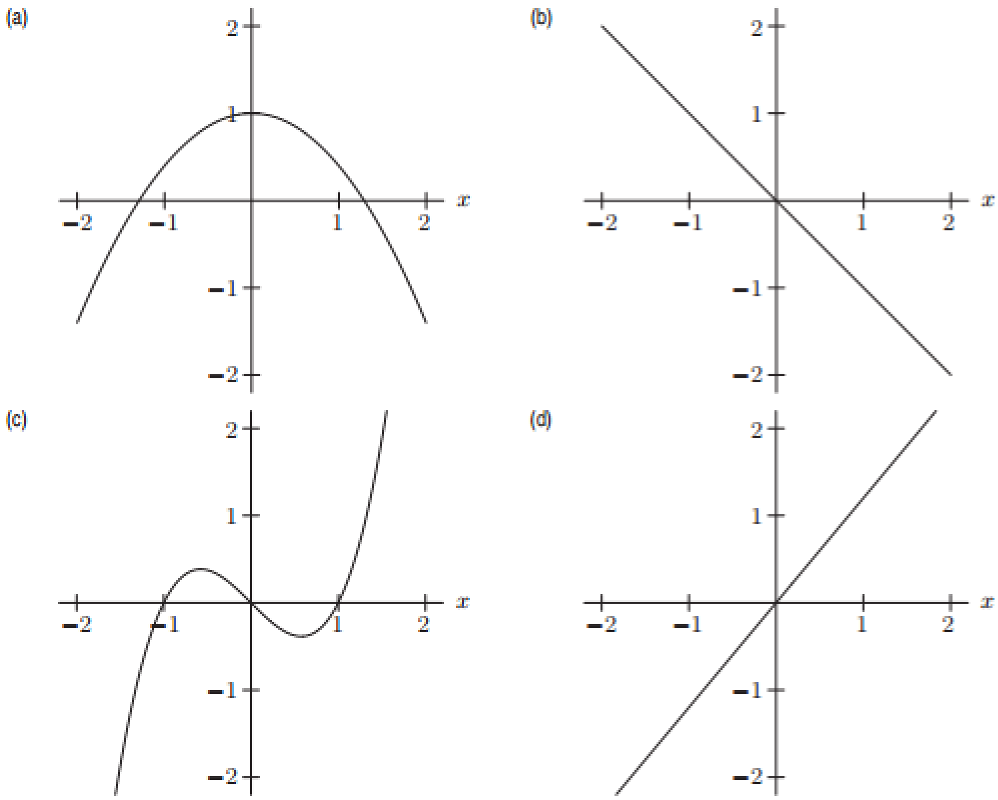


Figure 2.6



21.

Which of the following graphs (a)–(d) could represent the slope at every point of the function graphed in Figure 2.8?

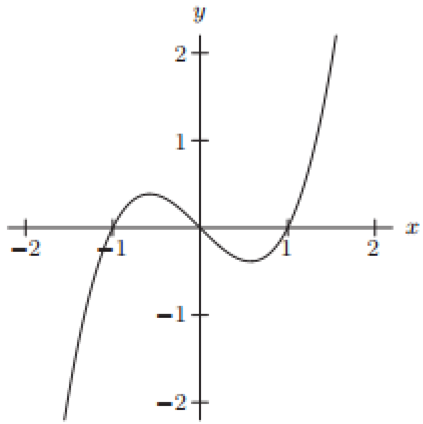
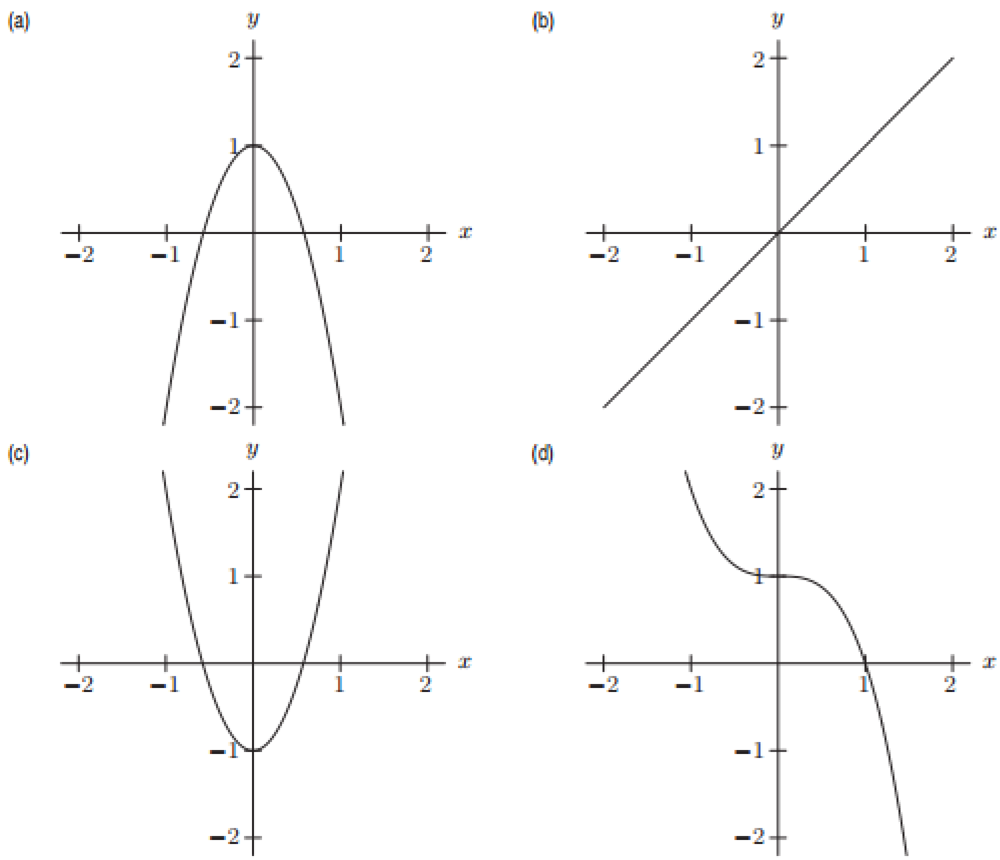


Figure 2.8



22.

Which of the following graphs (a)–(d) could represent the slope at every point of the function graphed in Figure 2.11?

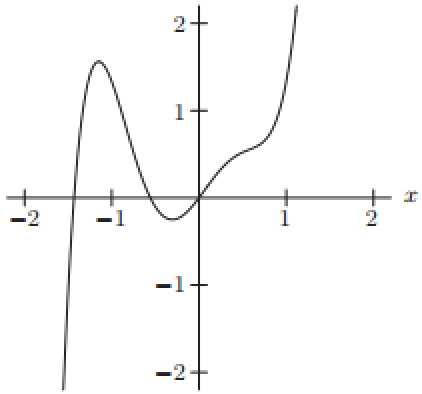
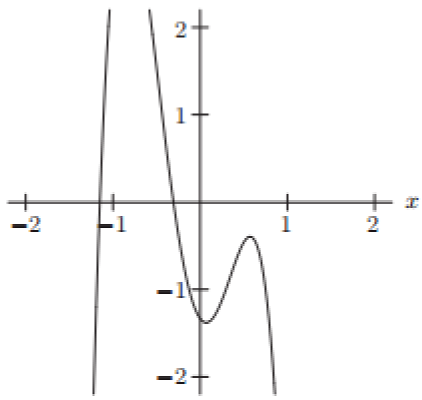
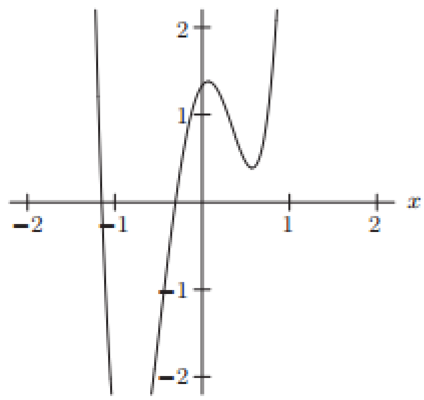


Figure 2.11

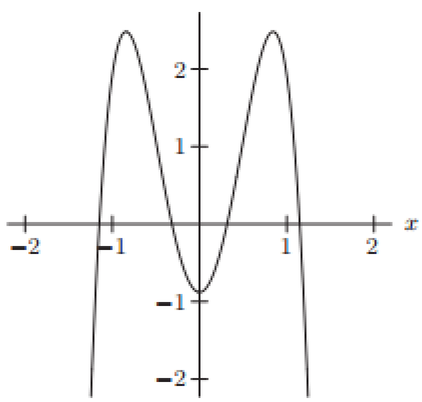
(a)



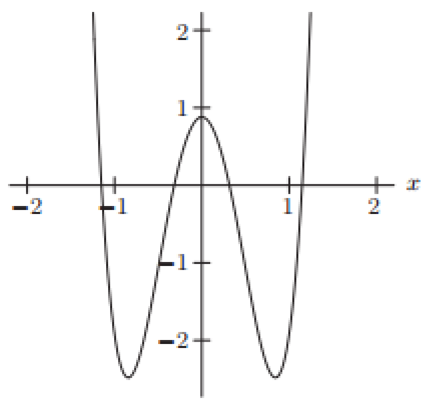
(b)



(c)



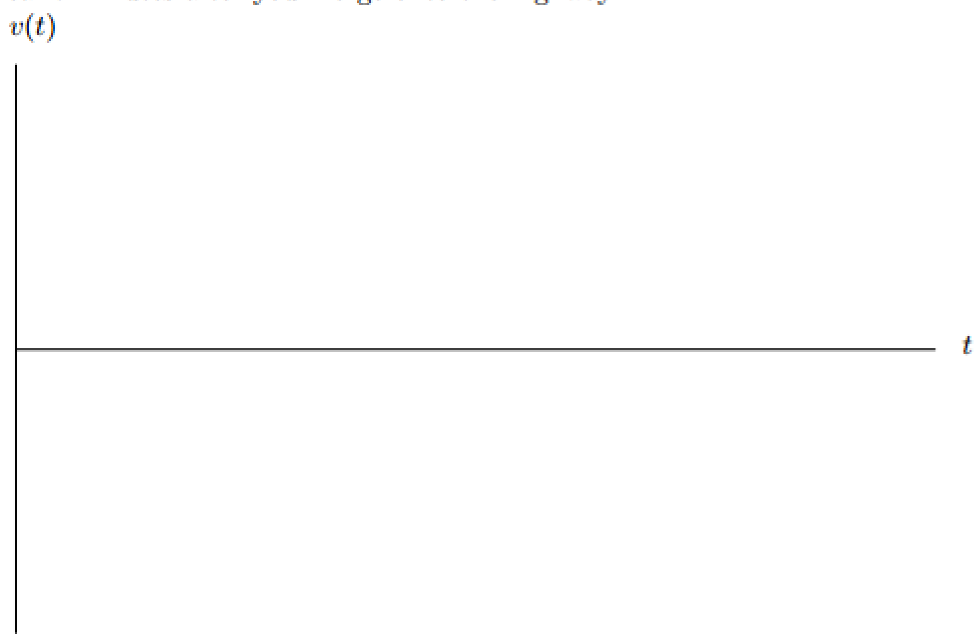
(d)



23.

[12 points] Suppose that when you merge onto the highway the blue car in front of you is moving at 55 mph. Immediately after you merge, the driver of the blue car speeds up until, after five minutes, it is going 85 mph. Then, during the next five minutes it slows down to 55 mph again. This process then repeats over the following 10 minutes, with the blue car speeding up to 85 mph and then decreasing to 55 mph again.

a. [6 points] Assuming the speed of the blue car follows a sinusoidal pattern, on the axes below draw a well-labeled sketch of two periods of a function $v(t)$ which outputs the speed of the car t minutes after you merge onto the highway.



What we call the beginning is often the end
And to make an end is to make a beginning.
The end is where we start from.

\- T. S. Eliot, *Little Gidding*