PRACTICE PROBLEMS FOR QUIZ IV

1. (a) Can you find a formula for d/dx (f (x) g(x) h(x))?

   Can you extend this to a product rule for four or more factors?

   (b) Using your result from (a), compute d/dx $\{5(x^3)(\cos x)(\ln x)\, e^x\}$

   (c) Find any and all critical points of the function: $y = (x^2 + 3)(x - 5)\, e^x$

2. Let $F(x) = x^3(x - 1)(x - 2)$.

   (a) Using precalculus techniques, sketch the curve y = F(x).

   (b) Using calculus, determine regions of increase, regions of decrease, local max/min.

   (c) Explain the geometric significance of each of the three *critical points*.

3. Let $g(x) = x^2 e^x$.

   (a) Using precalculus techniques, sketch the curve y = g(x).

   (b) Using calculus, determine regions of increase, regions of decrease, local max/min.

4. State carefully the *General Power Rule*. Review its proof, that depends only upon the product rule.

5. Using the General Power Rule, when appropriate, find dy/dx for each of the following functons:

   (a) $y = (1 + x + x^2)^{2015}$

   (b) $y = \sec^3 x$

   (c) $y = (e^x + 1)^{-3}$

   (d) $y = \sqrt{\dfrac{ax+b}{cx+d}}$ where *a*, *b*, *c* and *d* are non-zero constants. (Here you may assume that $d/dx\,(x^p) = px^{-1}$ for all real numbers p)

(e) $y = \tan^5 x$

(f) $y = (\sinh x + \cosh x)^{1789}$

(g) $y = \dfrac{1}{x^5 + 99}$

6. State the *Chain Rule*.

7. Explain why (d/dx) ln x = 1/x.

8. Using the Chain Rule, when appropriate, compute dy/dx for each of the following:

(b) $y = \sec(e^x + 4x + 1789)$

(c) $y = e^{\tan x}$

(d) y = ln(ax + b), where *a* and *b* are positive constants.

(e) $y = (\ln x)^4$

(f) $y = \ln(x^4)$

(g) y = ln(ln(x)

(h) y = ln(ln(lnx))

(i) y = cosh(sinh(3x+1))

(j) y = cos(sec(x))

(k) y = tan(1/x)

9. Let $y = u^3 + 1$ and u = 5 sin x. Using the chain rule, compute dy/dx

10. Let z = sin u and $u = 5 + e^x$. Compute dz/dx.

11. *Let* $f(x) = e^{x^2}$. Compute $f^{(2)}(x)$, the second derivative of *f*.

12. Let g(x) = cos(5x). Compute $g^{(2015)}(x)$.

13. (a) Given $y = \frac{\ln x}{x}$, compute d^2y/dx^2.

(b) Given $y = 5^x$, compute dy/dx.

(c) Given $y = \log_{13} x$, find dy/dx.

14. Sketch the graph of each of the following functions, using the first two stages of our plan.

Be sure to identify the domain first.

(a) y = x + 1/x

(b) $y = \sqrt{9 - x^2}$

(c) $y = x^{10} - 10x$

(d) $y = x^3 + 6x^2 + 1$]

(e) $y = x^3 + x^5 + x^7$

(f) $y = x^2 + \frac{1}{x^2}$

15. Using *implicit differentiation*, find dy/dx for each of the following implicitly defined curves:

(a) xy + x + y = y sin x

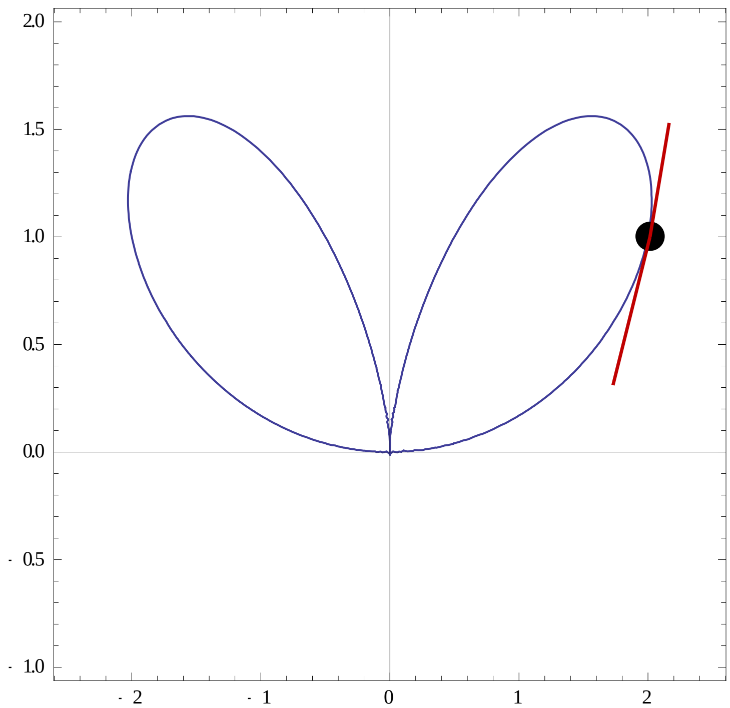
(b) tan x + sec y = x + y + 2015

(c) $xy^4 - \tan x = e^y + 1234$

16. Find an equation of the tangent line to the *bifolium*

$$4x^4 + 8x^2y^2 - 25x^2y + 4y^4 = 0$$

at the point P = (2, 1).



Who has not been amazed to learn that the function $y = e^x$, like a phoenix rising again from its own ashes, is its own derivative?

- Francois le Lionnais, ***Great Currents of Mathematical Thought***, *vol. 1*, Dover Publications

I turn away with fright and horror from the lamentable evil of functions which do not have derivatives.

- Charles Hermite (in a letter to Thomas Jan Stieltjes)