PRACTICE PROBLEMS FOR QUIZ V

1. Using *implicit differentiation*, find dy/dx for each of the following implicitly defined curves:

   (a) $xy + x + y = y \sin x$

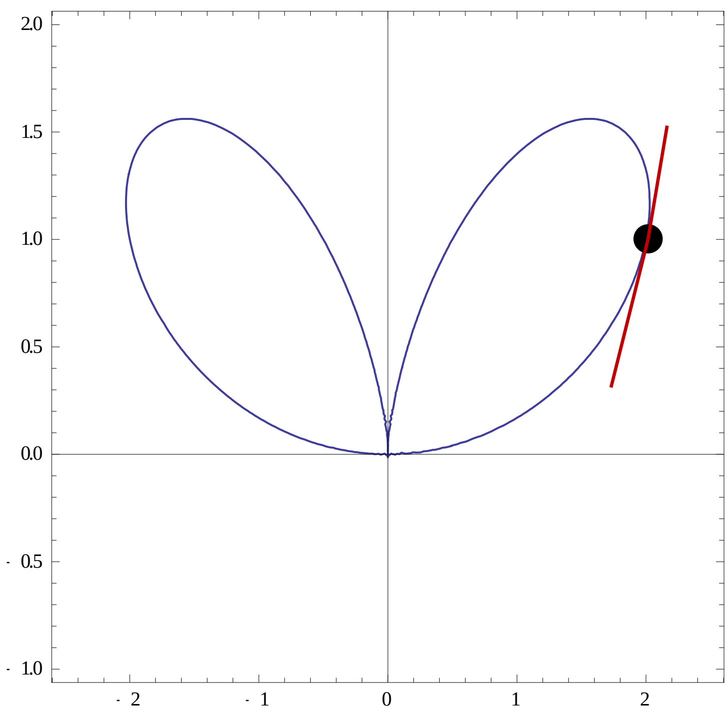
   (b) $\tan x + \sec y = x + y + 2015$

   (c) $xy^4 - \tan x = e^y + 1234$

2. Find an equation of the tangent line to the *bifolium*

$$4x^4 + 8x^2y^2 - 25x^2y + 4y^4 = 0$$

   at the point P = (2, 1).



3. Using implicit differentiation derive the formula for the derivative of each of the following functions:

4. Differentiate each of the following functions:

(a) $y = x^x$

(b) $y = (\sin x)^{\ln x}$

(c) $y = \arcsin(3x)$

(d) $y = \text{arcos}(5x – 13)$

(e) $y = (\text{arcsec } x) / x$

(f) $y = \arctan x + 3 \arcsin x$

(g) $y = \arctan((x – 1)/(x + 1))$

(h) $y = \arctan\left(\frac{1}{x}\right)$

(i) $y = \left(\arcsin\left(x^2\right)\right)^5$

5. Match the parameterizations (a)–(e) and curves I–V.

I. Line
II. Circle
III. Right angle
IV. Acute angle
V. Wave

(a) $x = \sin t$, $y = -t$
(b) $x = |3t|$, $y = t$
(c) $x = \sin t$, $y = \cos t$
(d) $x = 3t^3$, $y = t^2$
(e) $x = |3t|$, $y = 3t$

6. The equation of the line tangent to the curve $x = \sin 3t$, $y = \sin t + 1$ at the point where $t = 0$ is

(a) $y = 3x$
(b) $y = 3x + 1$
(c) $y = (1/3)x$
(d) $y = (1/3)x + 1$
(e) none of the above

7. (a) Find parametric equations and a parameter interval for the motion of a particle that starts at (5, 0) and traces the ellipse $x^2/25 + y^2/81 = 1$ twice counterclockwise.

(b) Find a parameterization of the line segment with endpoints (-1, -3) and (4, 1).

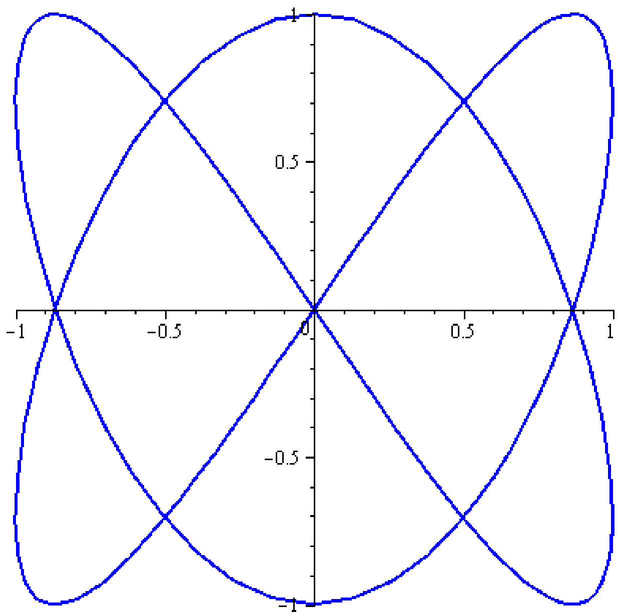
8. The figure below is called a Lissajous figure. (Lissajous curves have applications in physics, astronomy, and other sciences.) It is parameterized by the equations:

$x(t) = \sin 2t$

$y(t) = \sin 3t$

(a) Find the point in the interior of the first quadrant where the tangent to the curve is horizontal.

(b) Find the equations of the two tangent lines at the origin.



9. Using implicit differentiation, find dy/dx for

y = arc sin x, y = arc tan x, and y = arc sec x.

10. Find dy/dx for each of the following:

1. $y = \arcsin(2x+5)$

2. $y = \arctan\left(\frac{1}{x}\right)$

3. $y = \ln(arc\sec x)$

4. $y = \left(\arcsin(x^2)\right)^5$

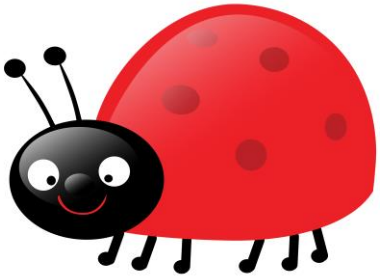
11. Using logarithmic differentiation, find dy/dx for each of the following:

1. $y = x(2x+1)^5(3x-4)^{13}$

2. $y = x\left(\frac{5x+12}{\sqrt{17x+5}}\right)$

3. $y = \sqrt{\frac{x(3x+1)(2x+5)}{(5x-1)(4x-8)}}$

12. Find parametric equations for a line through the points, $A = (-1, 1)$ and $B = (2, 3)$ so that the point A corresponds to $t = 0$ and the point B to $t = 6$.
The equations are x = ______ and y = ______, for $-\infty < t < \infty$.

13.  A lady bug moves on the *xy*-plane according the equations

$$x = 2t(t-6), \qquad y = 5-t.$$

When does the lady bug stop moving?

A) When $t = 6$ B) When $t = 3$ C) When $t = 5$ D) Never

14. A lady bug moves on the *xy*-plane according the equations

$$x = 2t(t-8), \qquad y = 2-t.$$

When is the lady bug ever moving straight up or down?

A) When $t = 8$ B) When $t = 4$ C) When $t = 2$ D) Never

15. A lady bug moves on the *xy*-plane according the equations

$$x = 2t(t-8), \qquad y = 3-t.$$

Suppose that the temperature at a point (x, y) in the plane depends only on the y coordinate of the point and is equal to $4y^2$. Find the rate of change of the temperature at the location of the lady bug at time t.

16. The equations $x = \frac{2}{\pi}\cos(\pi t/180),\ y = \frac{2}{\pi}\sin(\pi t/180)$ describe the motion of a particle moving on a circle. Assume x and y are in miles and t is in days. What is the radius of the circle (in miles)? Round to 2 decimal places.

17. The equations $x = \frac{3}{\pi}\cos(\pi t/90),\ y = \frac{3}{\pi}\sin(\pi t/90)$ describe the motion of a particle moving on a circle. Assume x and y are in miles and t is in days. What is the *period* of the circular motion (in days)?

18. The equations $x = \frac{9}{\pi}\cos(\pi t/180),\ y = \frac{9}{\pi}\sin(\pi t/180)$ describe the motion of a particle moving on a circle. Assume x and y are in miles and t is in days. What is the speed of the particle (in miles per day)

when it passes through the point ($-9/\pi$, 0)? Round to 3 decimal places.

19. Which of the following parametric equations pass through the points (8, 1) and (–1, –1)?

A) $x = 8 - 9t$ and $y = 1 - 2t$

B) $x = -1 - 9t$ and $y = -1 - 2t$

C) $x = 8 + 9t$ and $y = 1 + 2t$

D) $x = -1 + 9t$ and $y = -1 + 2t$

Mathematics compares the most diverse phenomena and discovers the secret analogies that unite them.

- Jean Baptiste Joseph Fourier (1768-1830)