Practice Problems for Quiz VIII

1. (a) Carefully state Rolle’s Theorem.

(b) Carefully state the Mean Value Theorem.

(c) Define the function G on the interval [-1, 2] as follows:



(d) Explain why *G* satisfies the hypotheses of the Mean Value Theorem on the interval [-1, 2]. Sketch!

(e) Determine the value of *c* for the function *G* on the interval [-1, 2] that is guaranteed by the Mean Value Theorem.

1. Solve each of the following initial value problems:





1. Solve the initial value problem:

 given that y = 2014 when x = 1.

4. Solve the differential equation:



5. (a) State **Rolle’s Theorem**.

1. Using Rolle’s Theorem, prove that the function

g(x) = (x – 2) ln (x + 1) + x sin(4x)

has at least one critical point between x = 0 and x = 2? Explain!

6. (a) State the **Mean Value Theorem**.

(b) Show how the Mean Value Theorem applies to the function f(x) = 4 + ln x on the interval [1, e3]. Sketch! Find explicitly the *c* value.

7. Solve the initial value problem:

 given that y = 2014 when x = 1.

8. Solve the differential equation:



9. Verify the formula (by differentiating):



10. *Verify* the following anti-differentiation formula:



11. Find an anti-derivative of each of the following functions. Show your work!







*Hint:* Express this as a function of sin x multiplied by cos x.

 *Hint:* Similar to (c).

12. Find the *indefinite integral* of each of the following functions. Show your work!







13. Solve the following initial value problem:

 given that y = 3 when t = 0.

14. The figure below is called a [Lissajous figure](http://en.wikipedia.org/wiki/Lissajous_curve). (Lissajous curves have applications in physics, astronomy, and other sciences.) It is parameterized by the equations:

x(t) = sin 2t

y(t) = sin 3t

1. Find the point in the interior of the first quadrant where the tangent to the curve is horizontal.
2. Find the equations of the two tangent lines at the origin.



15. Below is the graph of the derivative, F′(x), of a function F(x).

1. Sketch the graph of F′′[x].
2. Sketch the graph of F[x]. Indicate local max/min, regions of increase/decrease, regions where *F* is concave up/down, and all inflection points.



16. Using the method of *judicious guessing*, find an anti-derivative of each of the following functions:

1. 
2. cosh x – 3 sin 3x + sec2 (4x)
3. ex – e4
4. 
5. 
6. F(x) = a x4 + b sin cx
7. F(x) = x sin(x2) + 1
8. F(x) = sec2(x) + (sec x)(tan x)
9. F(x) = 9 cosh x
10. F(x) = sin(4x) + e9x
11. F(x) = x4(3 + 4x5)6
12. 
13. 

17. Can the following limit be solved using L’Hôpital’s rule? Explain.



18. Can the following limit be solved using L’Hôpital’s rule? Explain.



19. Can the following limit be solved using L’Hôpital’s rule? Explain.



20. Can the following limit be solved using L’Hôpital’s rule? Explain.



21. Can the following limit be solved using L’Hôpital’s rule? Explain.



22. *(MIT 18.01 final)*

Use L’Hôpital’s rule to compute the following limits:

1. 
2. 

23. Using de l’Hôpital’s rule, compute the following limit:



*A man is like a fraction whose numerator is what he is and whose*

*denominator is what he thinks of himself. The larger the*

*denominator, the smaller the fraction.*

– Tolstoy