# Practice Problems for TEST III

1. Find an anti-derivative of each of the following functions. Show your work!







*Hint:* Express this as a function of sin x multiplied by cos x.

 *Hint:* Similar to (c).

2. Find the *indefinite integral* of each of the following functions. Show your work!







3. Using de l’Hôpital’s rule, compute the following limit:



4. Solve the following initial value problem:

 given that y = 3 when t = 0.

5. We wish to approximate the value of



1. Sketch the curve *g(x) = 1 – 2-x* over the interval [1, 5].

Using *two rectangles* of equal base length, compute each of the following estimates:

(b) left-hand endpoints

(c) right-hand endpoints

1. midpoints

6. *Verify* the following anti-differentiation formula:



7. Using only the area interpretation of the definite integral, compute each of the following. (Sketch!)







8. Compute each of the following. *Simplify* your answers as much as possible.













9. Evaluate



by interpreting this definite integral in terms of areas. Sketch!

10. Consider the graph of y = f ′(x) drawn below. (*Note: This is not the graph of f.*)



1. On which intervals, if any, is *f*  *increasing?*

(b) At which values of *x* does *f* have a *critical point?*

(c) On which intervals, if any, is *f* *concave up?*

(d) Which values of *x*, if any, correspond to *inflection points* on the graph

of *f*?

1. Assume that f(0) = 0. Sketch a graph of *f*. (Your graph need only have the right general shape. You do not need to put units on the vertical axis.)

11. State the *two versions* of the Fundamental Theorem of Calculus.

12. Albertine has purchased a Chevy Volt that can accelerate from 0 ft/sec to 88 ft/sec in 5 seconds. The car’s velocity is given below:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| *t (seconds)* | 0 | 1 | 2 | 3 | 4 | 5 |
| *v(t) (ft/second)* | 0 | 30 | 52 | 68 | 80 | 88 |

Using five rectangles, find upper and lower bounds (that is, over and under-estimates) for the distance traveled by Albertine’s car in 5 seconds.



13. Using the FTC, evaluate each of the following. You *need not* simplify your answers.







14. Using the method of judicious guessing evaluate:



15. Using the FTC, compute g'(x) given that



What is the value of g'(e) ?

16. Find an anti-derivative of each of the following functions. Show your work!

(a) tan x

(b) tan2 x

(c) sin2 x

*Hint:* Recall that 

(d) cos2 x

*Hint:* Recall that 

(e) sec4 x

*Hint:* Write sec4 x = (sec2 x)(sec2 x) and then apply a basic identity.

17. Solve the initial value problem:



given that y(1) = 13.

18. Evaluate each of the following indefinite integrals:

* 1. 
  2. 
  3. 

19. Evaluate each of the following definite integrals using only the geometric interpretation of the definite integral. Explain your solution.

(a) 

(b) 

(c) 

(d) 

20. Calculate each of the following sums. Simplify each answer.

(a) 

(b) 

(c) 

|  |
| --- |
|  |
| |  | | --- | |  | |
|  |

21. Given that 

compute the value of 

22. Find the *average value* of the function f(x) = 4 sin 2x over the interval [0, ].

23. Which values of *a* and *b* *minimize* the value of



24. Find dy/dx given that



25. Find the area bounded by the x-axis and the curve y = x(x – 3)2.

26. Find the area between the curves y = 5 – x and y = 7 – x2.

27. Find the area of the region in the first quadrant that is bounded above by y = x1/2 and below by the x-axis and the line y = x – 2.

28. Compute the *average value* of the curve y = cos2 4x over the interval [0, ].

29. Let f(x) be a continuous function. Express



as a definite integral.

30. Compute the value of the following limit



by expressing this limit as a Riemann integral and then evaluating the integral.

31. Evaluate each of the following.

(a) 

(b)  *(and simplify)*

(c)  *(and simplify)*

(d) 

32. Consider the following table of data:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| x | -1.00 | -0.25 | 0.50 | 1.25 | 2.00 |
| F(x) | 0.0000 | 2.6522 | 4.8755 | 6.8328 | 8.6790 |

Approximate the area below the graph of y = F(x) above the interval [-1, 2] using:

(a) left endpoints. *Sketch.*

(b) right endpoints. *Sketch.*

33. Suppose that Madame Verdurin wishes to find a solution to the equation

ln x = x – 5.

(a) Sketch the graphs of the two curves (y = ln x and y = x – 5) on the same pair of axes. How many solutions would you expect to find?

(b) Define a function, G(x), whose roots will provide Verdurin with the answer to her quest.

(c) Assume that Verdurin’s first guess is x0 = 6. Verdurin uses Newton’s method to compute x1 and x2. Which values should she obtain! Show your computation.

34. Solve the following initial value problem:



given that y(0) = 11.

35. Verify the following integration formula:



36. At time *t*, in seconds, the velocity, *v*, in miles per hour, of Albertine’s new Prius is given by

v(t) = 5 + 0.8t2  for 0 < t < 8.

Use Δt = 2 to estimate the distance traveled during this time. Find the left endpoint and right endpoint sums, and the average of the two. *Sketch!*

37. Using the method of judicious guessing, find an *antiderivative* for each of the following functions. *Be certain to show your reasoning!*

(a) 

(b) 

(c) 

(d) 

38. Suppose that  Evaluate 

39. Find the area of the region enclosed by y = x(a2 – x2)1/2 , where a is a positive constant and the y-axis.

40. Find the area of the region enclosed by y = (|x|)1/2 and 5y = x + 6.

41. Find the area of the region enclosed by x = y3 – y2 and x = 2y.

42. Find the area of the region enclosed by y = 2 sin x and y = sin (2x), 0 ≤ x ≤ .



### 44. Evaluate the following limit:



### 45. Evaluate the following limit



46. Evaluate the following limit



47. Evaluate the following limit



48. Evaluate the following limit



49. Explain why the value of cannot equal 4.5.

50. *[U. Michigan]* Using the graph of y = h(x) shown below, compute each of the following quantities. If there is not enough information to compute the given quantity write “not enough information.” Justify each answer!









*(D)* The *average value* of h(x) on the interval [-2, 2], assuming that h(x) is an even function.

*(E)* H(2), where H is an anti-derivative of *h*.

*(F)* H(2) – H(0), where H is an anti-derivative of *h*.

51. (a) State **Rolle’s Theorem**.

1. Using Rolle’s Theorem, prove that the function

g(x) = (x – 2) ln (x + 1) + x sin(4x)

has *at least one* critical point between x = 0 and x = 2? Explain!

52. (a) State the **Mean Value Theorem**.

(b) Show how the Mean Value Theorem applies to the function

*f(x) = 4 + ln x* on the interval [1, e3]. Sketch! Find explicitly the *c* value.

53. Explain why any two anti-derivatives of a function F(x) must differ by a constant.

54. Find dy/dt given that



55. (a) State **Rolle’s Theorem**.

(b) Using Rolle’s Theorem, prove that the function

g(x) = (x – 2) ln (x + 1) + x sin(4x)

has *at least one* critical point between x = 0 and x = 2? Explain!

56. *(U. Michigan)* A car, initially going 100 feet per second, brakes at a constant rate (constant negative acceleration), coming to a stop in 8 seconds. Let t be the time in seconds after the car started to brake.

(a) Sketch a graph of the velocity of the car from t = 0 to t = 8, being sure to include labels.

(b) Exactly how far does the car travel? Make it clear how you obtained your answer.

57. *(U. Michigan)* Suppose dg/dx > 0 on the interval [3, 5], g(3) = 12, and g(5) = 20. We want to use a *Riemann sum* with equal-size subdivisions to approximate



If we want to guarantee that the error in our estimate is *no larger than* *¼*, then what is the minimum number of subdivisions that we must use?

58. *(U. Michigan)* If



find the value of



1. State *three corollaries* to the Mean Value Theorem.

60. *(Apostol)* A function *f* is continuous everywhere and satisfies the equation



for all x. Compute f(/4) and df/dx at x =  /4.

61. *(Apostol)* Find a function *f* and a value of the constant *c* such that, for all real *x*,



62. Can the following limit be solved using L’Hopital’s rule? *Explain.*



63. Can the following limit be solved using only L’Hopital’s rule? *Explain.*



64. Find:



65. Can the following limit be solved using L’Hopital’s rule? *Explain.*



66. Can the following limit be solved using only L’Hopital’s rule? *Explain.*



67. Can the following limit be solved using only L’Hopital’s rule? Explain.



68. *(MIT 18.01 final)*

Use L’Hopital’s rule to compute the following limits:





69. *(U. Michigan)* The rate at which a coal plant releases CO2 into the atmosphere *t* days after 12:00 am on January 1, 2015 is given by the function *E(t)* measured in tons per day. Suppose that 

(a) Give a practical interpretation of 

(b) Give a practical interpretation of E(15) = 7.1.

(c) The plant is upgrading to “clean coal” technology which will cause its July 2015 CO2 emissions to be one fourth of its January 2015 CO2 emissions. How much CO2 will the coal plant release into the atmosphere in July?

(d) Using a left-hand sum with four subdivisions, write an expression which approximates 

70. *(U. Michigan)* Shown below is a graph of a function f(t). The graph consists of a straight line between t = 0 and t = 2 and a quarter circle between t = 2 and t = 3.



Calculate the following using the graph and the properties of integrals.

1. 
2. The *average* *value* of *f* on the interval [1, 3].

71. *(U. Michigan)* A car, initially going 100 feet per second, brakes at a constant rate (constant negative acceleration), coming to a stop in 8 seconds. Let t be the time in seconds after the car started to brake.

(a) Sketch a graph of the velocity of the car from t = 0 to t = 8, being sure to include labels.

(b) Exactly how far does the car travel? Make it clear how you obtained your answer.

72. *(U. Michigan)* Suppose dg/dx > 0 on the interval [3, 5], g(3) = 12, and g(5) = 20. We want to use a Riemann sum with equal-size subdivisions to approximate



If we want to guarantee that the error in our estimate is no larger than ¼, then what is the *minimum* number of subdivisions that we must use?

*Learning without thought is labor lost; thought without learning is perilous.*

- Confucius