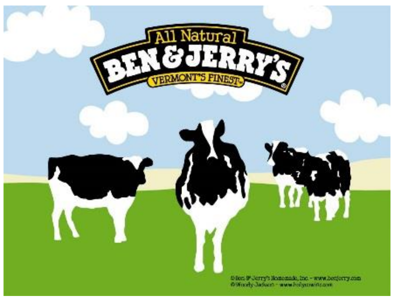
MATH 161 - TEAM HOMEWORK ASSIGNMENT I

(an adaptation of a University of Michigan problem)

- Due Date: Thursday, Oct 8th, at 5 pm
  Either give your final writeup to me in my office or place in my mailbox on the 5th floor of BVM
- Remember to follow the guidelines from the "Doing Team Homework." Do not forget to submit the "reporter's page" with the assignment
- Please show ALL work.



1. Albertine and Gilberte are freshmen roommates at the Loyola University. Both are Ben and Jerry's ice cream lovers, so naturally they are curious about the relationship between the average monthly temperature and the amount of ice cream they consume that month. They have made several measurements, which appear in the table below. A (respectively G) is Albertine's (respectively Gilberte's) consumption of ice cream (in gallons, measured to the nearest tenth of a gallon) in a month when the average monthly temperature is T (in degrees Fahrenheit, measured to the nearest degree).

$T(^\circ F)$	A (gallons)	G (gallons)
30	0.9	1.1
70	1.6	1.7
90	2.1	2.3

(a) Based on this data, could either student's ice cream consumption be reasonably modeled as a linear function of temperature? An exponential function? Neither? Carefully justify your answer. (Hint: *At least* one student's ice cream consumption can be modeled by a linear or an exponential function!)

(b) Using your answer to part (a), predict how much ice cream *one* of the students will consume in March, when the average temperature is usually around $45^\circ F$. (If you were able to model both students' ice cream consumptions as exponential or linear, choose one student about whom to make the prediction. If you could model just one student's consumption, predict that student's consumption.)

(c) Gilberte and Albertine's friend Odette decides that she wants to join in on the fun and measure *her* ice cream consumption as a function of temperature. Odette is from Australia, so she measures temperature in degrees Celsius, and she measures ice cream consumption in liters. She ends up with a function $m(t)$ which is the number of liters of ice cream she consumes in a month when the average monthly temperature is t degrees Celsius. If $M(T)$ is the number of *gallons* Gilberte consumes when the average monthly temperature is T degrees *Fahrenheit*, write a formula for $M(T)$ in terms of m and T. (You may need to look up some unit conversions online.)

2. Let $f(x) = 3-x$ and suppose $g(x)$ is a function whose domain is all real numbers. Functions h, u, and v are defined as follows:

$$h(x) = f(g(x)), \qquad u(x) = g(f(x)), \quad \text{and} \qquad v(x) = f(x)g(x).$$

(a) Some values of the functions g, h, u, and v are given in the table below. Fill in the missing values in the table. *Remember, as always, to explain your reasoning clearly, assuming your audience is a classmate who has never thought about this problem.*

x	$g(x)$	$h(x)$	$u(x)$	$v(x)$
0				
1	-4			
2				3
3		-6	10	

(b) Assume that $g(x)$ is constant for $x \leq 0$, linear between $x = 0$ and $x = 1$, linear between $x = 1$ and $x = 2$, exponential between $x = 2$ and $x = 3$, and constant again for $x \geq 3$. Sketch a graph of $g(x)$. Then use this graph to sketch graphs of the functions $h(x)$ and $u(x)$.

(c) Using the information from part (b), write a piecewise-defined formula for $g(x)$.

3. (a) Françoise began the school year with a total savings of \$100, and during the semester she works at Metropolis where she earns \$200 a week, all of which she is able to save. Let $F(t)$ be her total savings (in dollars) t weeks since the start of the semester. Assuming she stores all her money under her pillow, write a formula for $F(t)$.

(b) Swann began the school year with \$800 dollars, and although he was not working during the semester, he managed to find a bank that offered a savings account that pays 11.4% interest every 30 days. (He had to promise to keep the bank's location secret.) Let $S(t)$ be his total savings (in dollars) t weeks since the start of the semester. Write a formula for $S(t)$.

(c) Who has more money at the start of the semester? Does that student stay in the lead all semester? If not, during which week does the (first, if there is more than one) lead change occur?

(d) If both students continue saving according to the formulas in (a) and (b) forever, who will eventually have more money?

(e) Draw a well-labeled graph of the two functions that clearly indicates your answers to (c) and (d), as well as all times that both students have the same amount of money.

(f) Explain what the expression

$$F^{-1}\left(\frac{S(52)}{2}\right)$$

represents in the context of this problem. (*Remember to use a complete sentence, and include units.*)