Math 115 - Team Homework Assignment #4, Fall 2015

- **Due Date:** October 29 or 30 (Your instructor will tell you the exact date and time.)
- Note: All problem, section, and page references are to the course textbook, which is the 6th edition of *Calculus: Single Variable* by Hughes-Hallett, Gleason, McCallum, et al.
- Remember to follow the guidelines from the "Doing Team Homework" and "Team HW Tutorial" links in the sidebar of the course website.
- Do not forget to rotate roles and include a reporter's page each week.
- Show ALL your work.

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1. On a small planet in a galaxy far far away, there is a town called Srebmun Foyoj, where the company Sulu Clac Evoli (usually referred to as just "Clac") has a monopoly on the supply of coal. This means that Clac gets to choose a quantity Q of coal to produce, and the price P at which the coal sells depends on this quantity. In other words, $P = f(Q)$ for some function f. Suppose that Clac's revenue R is given by $R = Q \cdot P = Q \cdot f(Q)$, where Q is measured in tons of coal and $f(Q)$ is measured in dollars per ton. Some values of f and f' are shown in the table below.

Q	5	10	15	20	25	30	35
$f(Q)$	64.57	48.64	41.72	40.04	39.87	37.44	29.02
$f'(Q)$	−4.34	−2.16	−0.74	−0.06	−0.14	−0.96	−2.54

   (a) Find the value of $\left.\dfrac{dR}{dQ}\right|_{Q=35}$ and give a practical interpretation of your answer in the context of this problem.

   (b) Evita Vired is an economic consultant on Srebmun. Clac goes to her with the values of $f(35)$, $f'(35)$, and $\left.\dfrac{dR}{dQ}\right|_{Q=35}$ and asks what they should do to increase revenue without changing production by more than a few tons. Should she advise the company to produce a few more tons, a few less tons, or the same number of tons? Explain your reasoning.

   (c) Estimate the value of $\left.\dfrac{d^2R}{dQ^2}\right|_{Q=15.}$

2. Consider the parabola given by the equation $w(x) = x^2 - 5x + 9$.

   (a) Find the equation of the tangent line to this parabola at the point $(1, 5)$.

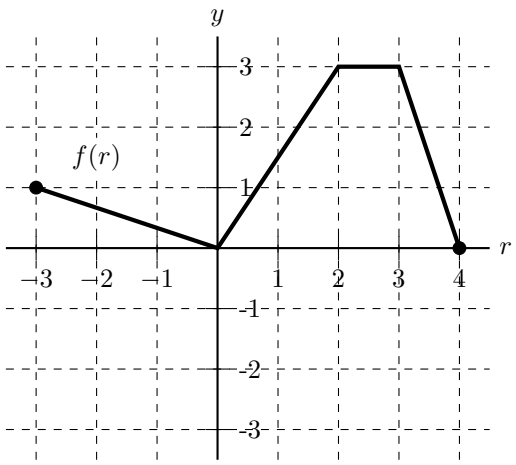
   (b) Sketch a graph of the parabola and the tangent line from part (a).

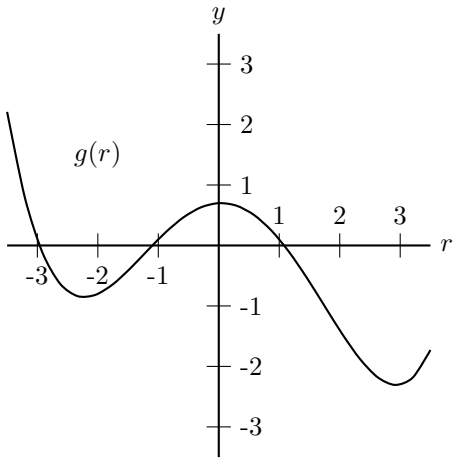
   (c) Use your graph to predict how many lines tangent to this parabola pass through the origin. Draw a new graph of the parabola and illustrate your answer (i.e., draw in the tangent lines that you think pass through the origin).

   (d) Use calculus to find the equations of all the lines tangent to this parabola that pass through the origin. Compare your answers with your prediction in part (c).

There is another problem on the next page.

3. The graphs of two functions, $f(r)$ and $g(r)$, are shown below.





The following questions concern the functions h, p, and s defined as follows:

$$h(r) = \frac{f(r)}{f(2r)}, \qquad p(r) = f(f(r)), \quad \text{and} \quad s(r) = e^{-g(r)}.$$

Assume that the first and second derivatives of $g(r)$ are defined everywhere, i.e. that g and g' are differentiable on $(-\infty, \infty)$. Note that the graph of $f(r)$ consists of line segments whose endpoints have integer (whole number) coordinates.

(a) For each of the following expressions, either find the exact value or explain why the expression is undefined.

i. $h'(1.6)$

ii. $h'(1)$

iii. $p'(-1)$

(b) On the interval $[0.5, 2]$, is $s(r)$ always increasing, always decreasing, or neither? Explain your reasoning.

(c) On the interval $[-0.5, 0.5]$, is $s(r)$ always concave up, always concave down, or neither? Explain your reasoning.