Math 115 - Team Homework Assignment #6, Fall 2015

- **Due Date:** November 12 or 13 (Your instructor will tell you the exact date and time.)
- Note: All problem, section, and page references are to the course textbook, which is the 6th edition of *Calculus: Single Variable* by Hughes-Hallett, Gleason, McCallum, et al.
- Remember to follow the guidelines from the "Doing Team Homework" and "Team HW Tutorial" links in the sidebar of the course website.
- Do not forget to rotate roles and include a reporter's page each week.
- Show ALL your work.

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1. Consider the function $g(x) = \dfrac{12}{3+x^2}$.

   (a) Let $L(x)$ be the local linearization of $g(x)$ at $x = 3$. Find a formula for $L(x)$. Sketch graphs of the functions $L(x)$ and $g(x)$ on the same set of axes.

   (b) Using $L(x)$, approximate $g(3.2)$. Similarly, approximate $g(2.8)$. Are your estimates overestimates or underestimates?

   Recall that for a function $f(x)$ we define "the quadratic approximation"[1] of $f(x)$ at $x = a$ to be the degree 2 polynomial $Q(x)$ such that all of the following hold:

   - $Q(x)$ and $f(x)$ have the same function value at $x = a$.
   - $Q(x)$ and $f(x)$ have the same first derivative at $x = a$.
   - $Q(x)$ and $f(x)$ have the same second derivative at $x = a$.

   A formula for $Q(x)$ is given by

   $$Q(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2\,.$$

   (c) With $g(x)$ as above, let $P(x)$ be the quadratic approximation of $g(x)$ at $x = 3$. Find a formula for $P(x)$. Sketch graphs of the functions $g(x), L(x)$, and $P(x)$ on the same set of axes.

   (d) Using $P(x)$, approximate $g(3.2)$ and $g(2.8)$. Are your estimates overestimates or underestimates? Are they better or worse than your estimates from part (b)?

   (e) Justify the following statement: "For large values of x, $P(x)$ is not a good approximation of $g(x)$."

2. Consider the function

   $$f(x) = \begin{cases} \frac{1}{4}x^8 - x^6 - \frac{1}{4} & \text{for } x \le 1 \\ (x-2)^{\frac{1}{5}} & \text{for } x > 1. \end{cases}$$

   (a) Find all local extrema of $f(x)$. Classify each extremum as a local maximum or a local minimum. *Use calculus to find and justify your answer, and be sure to show enough evidence to demonstrate that you have found all local extrema.*

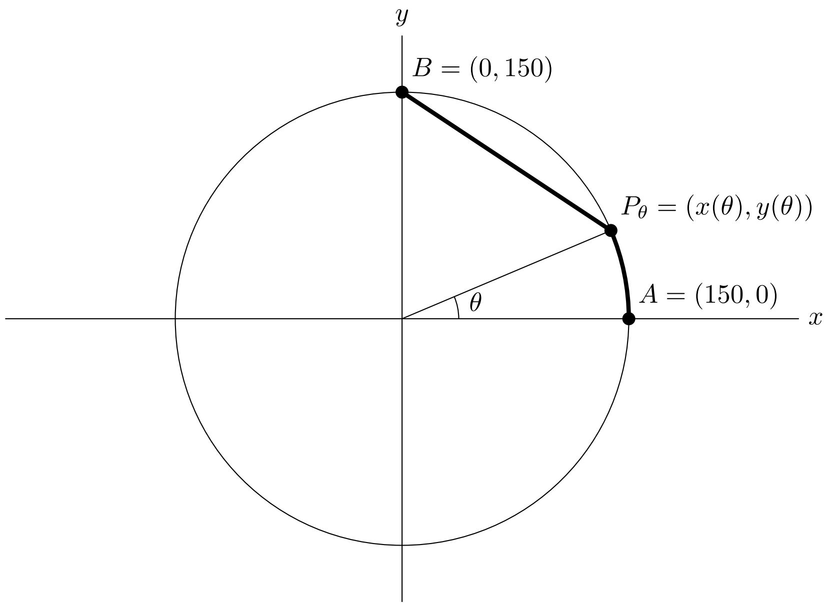
   (b) Find all inflection points of $f(x)$. *Use calculus to find and justify your answer, and be sure to show enough evidence to demonstrate that you have found all inflection points.*

   (c) Sketch a graph of $f(x)$ on the interval $-5 < x < 5$.

This assignment continues on the next page.

[1] This is the Taylor polynomial of degree 2 for $f(x)$ at $x = a$.

3. Maddy is standing at point A on a circular track, and there is a spaceship at point B waiting to take her to Srebmun Foyoj. She runs at a constant speed of 8 meters/second around the track, but only at a constant speed of 6 meters/second on the tall grass inside the track. The radius of the track is 150 meters.



Let $T = M(\theta)$ be the time (in seconds) that it takes Maddy to get from point A to point B if she runs counterclockwise along the track from A to P_θ, and then across the grass in a straight line from P_θ to B. (This is the path shown in bold in the diagram.)

(a) Write a formula for T in terms of θ. Your answer should **not** contain x or y. [You may find it helpful to recall that the arclength of the portion of a circle of radius r units which is swept out by an angle of α radians has length αr units.]

(b) Maddy can't wait to go explore a new planet, so she wants to minimize the time it takes her to get from point A to point B. Explain why she only needs to consider angles θ in the interval $0 \le \theta \le \pi/2$.

It can be shown that for $0 \le \theta < \pi/2$, the function from part (a) has the following derivative:

$$\frac{dT}{d\theta} = M'(\theta) = 150\left(\frac{1}{8} - \frac{\sqrt{1+\sin\theta}}{6\sqrt{2}}\right).$$

M is not differentiable at $\theta = \pi/2$. Use this information to answer parts (c) and (d).

(c) What value(s) of θ in the interval $0 \le \theta \le \pi/2$ will minimize Maddy's running time? *Use calculus to find and justify your answer.*

(d) Maddy learns that there is no ice cream in Srebmun Foyoj (as it is too hot there), so she is suddenly much less excited to go there. What value(s) of θ in the interval $0 \le \theta \le \pi/2$ will maximize her running time? Assume that her speed on each surface is the same as before. *Use calculus to find and justify your answer.*

(e) Cal is standing next to Maddy at point A, and he wants to get to the spaceship as quickly as possible (he missed the memo about ice cream). Unlike Maddy, Cal runs at a constant speed of 5 meters/second on both the track and the grass. What path minimizes the time it takes Cal to get to the spaceship? Is it the same path that Maddy takes to minimize her time?
You may choose to use physical/geometric reasoning rather than calculus to find and justify your answer on this part.