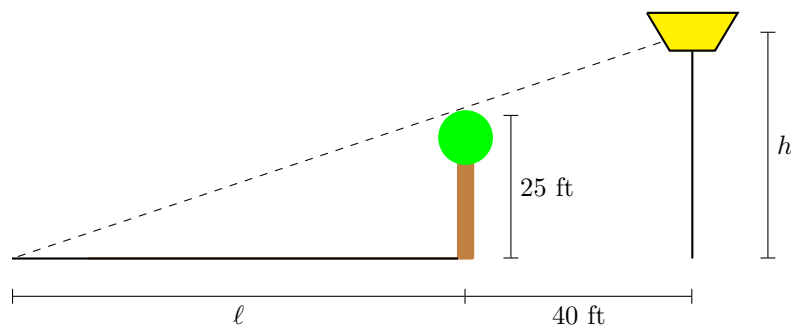


Math 115 - Team Homework Assignment #7, Fall 2015

- **Due Date:** December 9 or 10 (Your instructor will tell you the exact date and time.)
 - Note: All problem, section, and page references are to the course textbook, which is the 6th edition of *Calculus: Single Variable* by Hughes-Hallett, Gleason, McCallum, et al.
 - Remember to follow the guidelines from the “Doing Team Homework” and “Team HW Tutorial” links in the sidebar of the course website.
 - Do not forget to rotate roles and include a reporter’s page each week.
 - Show ALL your work.
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1. A tree blocks the light coming from a streetlamp, casting a shadow. The length ℓ of the shadow is a function of the height h of the lamp. The lamp is attached to a pole, which has a mechanism that changes its height over time. Between the hours of 12 pm and 6 pm, the height of the pole (in feet) is given by the function $h = u(t)$, where t is measured in hours after 12 pm. The table below gives several values of the functions u and u' . Assume the lamp is a horizontal distance of 40 feet from the tree, and that the tree is 25 feet tall. The diagram below depicts this scenario (note that it is not drawn to scale).

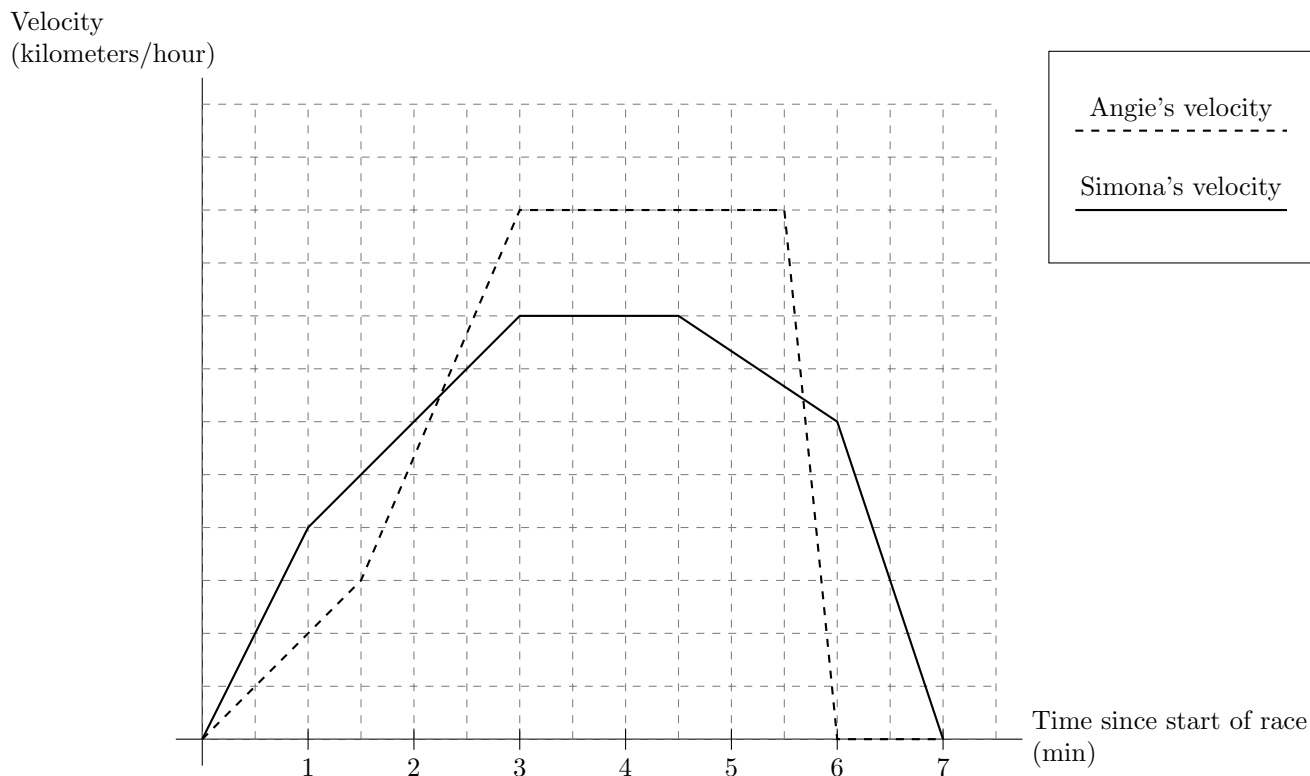


t	0	2	4	6
$u(t)$	70	67	57	41
$u'(t)$	0	-3.4	-6.5	-8.9

- (a) At 2 pm, what is the length of the shadow?
- (b) At 4 pm, how fast is the length of the shadow changing? Be sure to consider the meaning of the sign of your answer.

This assignment continues on the next page.

2. Angie and Simona run a 1500 meter race. After the race, Phil receives the following graph of the two runners' velocities over the course of the race:



Unfortunately, whoever made the graph forgot to label the scale of the vertical axis, and Phil needs your help to answer the following questions. You may assume that the horizontal grid lines are evenly spaced, but do not assume that the scales of the two axes are the same. Don't forget to explain your reasoning.

- Who won the race?
- At what times was Angie ahead of Simona?
- At what times was Angie running faster than Simona?
- What was the maximum speed (in kilometers per hour) attained by Angie? By Simona?
- Let $n(t)$ (respectively, $m(t)$) be Angie's (respectively, Simona's) velocity in kilometers per hour t minutes after the start of the race. Write an equation involving one or more integrals that expresses the following statement: p minutes after the start of the race, Angie is w kilometers ahead of Simona.

This assignment continues on the next page.

3. In order to share the delicious lava cake and the beautiful varieties of mumertxe from Srebmun Foyoj with their friends back on Earth (and to be able to get the ice cream they've been craving), Maddy and Cal open a spaceship company that ships cargo between Earth and Srebmun Foyoj. For each flight, their operating costs are \$10000 for fuel, and an additional \$40 for each pound of cargo on the spaceship. Each of their spaceships has a carrying capacity of at most 2 tons (4000 pounds) of cargo. They decide on the following pricing scheme: if there are no more than 1500 pounds of cargo, then they charge \$90 per pound. If there are more than 1500 pounds of cargo, then the price per pound (for all the cargo) decreases linearly at a rate of \$2 for every 100 pounds over 1500. For example, if there are 1550 pounds of cargo, then they charge \$89 for each pound.
- Find a formula for the cost C of operating a flight with m pounds of cargo.
 - Find a formula for the revenue R that Maddy and Cal bring in if they operate a flight with m pounds of cargo.
 - What is the maximum profit that Maddy and Cal can make on a flight? *Be sure to include enough evidence to justify that you have indeed found the maximum profit.*
 - Algie, seeing how much money his friends are raking in, decides to open his own shipping business. He has the same costs as Maddy and Cal, and the same carrying capacity per spaceship, but he uses a slightly different pricing scheme: if there are no more than 2000 pounds of cargo, then he charges \$110 per pound. If there are more than 2000 pounds of cargo, then the price per pound (for all the cargo) decreases linearly at a rate of \$4 for every 100 pounds over 2000. What is the maximum profit that Algie can make on a flight? *Be sure to include enough evidence to justify that you have indeed found the maximum profit.*
4. Consider the family of functions $f(x) = ax^2 - 2$ for nonzero constants a . In this problem, we will find the points on the parabola given by $f(x) = ax^2 - 2$ that are closest to the origin.¹
- Sketch a graph of the parabola $f(x)$ for $a = -1, 1/8, 1/2, 1$ (be sure to indicate which graph corresponds to which value of a). How does a affect the shape of the graph?
 - Based on your observations from part (a), does it seem as though the parabola's vertex will always be the closest point to the origin?
- For (c)-(e) below, consider the entire family of functions $f(x) = ax^2 - 2$ with $a \neq 0$.
- Let $d(x)$ be the distance between the origin and the point $(x, f(x))$ on the parabola. Write a formula for $d(x)$.
 - In this situation (and many others) it is easier to find the values of x that minimize the function $D(x) = (d(x))^2$ (the square of the distance) than the values of x that minimize $d(x)$ itself. Give a brief explanation of why the values of x that minimize $d(x)$ and the values of x that minimize $D(x)$ are the same in this situation.
 - Find the points on the parabola that are closest to the origin. Your answer may depend on a . (Note that in some cases there may only be one such point.) *Use calculus to justify your answer, and be sure to include enough evidence to demonstrate that the points you have found do indeed minimize the distance.* [Hint: The Second Derivative Test may be useful.]

This assignment continues on the next page.

¹Credit for this problem is due to Brooke Ullery.

5. (a) Use a left Riemann sum with 4 equal subintervals to estimate $\int_{0.5}^1 5e^{-x^2} dx$. Be sure to write out all the terms of the sum. Illustrate your answer on a graph. Is your estimate an overestimate, an underestimate, or neither?
- (b) Use a right Riemann sum with 4 equal subintervals to estimate $\int_{-3}^5 5e^{-x^2} dx$. Be sure to write out all the terms of the sum. Illustrate your answer on a graph. Is this estimate an overestimate, an underestimate, or neither?
- (c) Consider the integral $\int_a^b 5e^{-x^2} dx$. Let $\text{LEFT}(n)$ denote the left Riemann sum estimate of this integral with n equal subintervals, and let $\text{RIGHT}(n)$ denote the right Riemann sum estimate of this integral with n equal subintervals. For each of the following, find values of a and b , with $a < b$, so that the statement is true for all positive integers n , and explain why your choice of a and b works.
- $\text{LEFT}(n) > \text{RIGHT}(n)$
 - $\text{LEFT}(n) < \text{RIGHT}(n)$
 - $\text{LEFT}(n) = \text{RIGHT}(n)$.