WORKSHEET I (CHALLENGING ALGEBRA PROBLEMS FROM H..L*)

(A) Solve for x:

1. $\sqrt{x+5}+\sqrt{3x+4}=\sqrt{12x+1}$

2. $6x^{\frac{3}{4}}=7x^{\frac{1}{4}}-2x^{-\frac{1}{4}}$

3. $\dfrac{x-8}{x-10}-\dfrac{x-5}{x-7}=\dfrac{x-7}{x-9}-\dfrac{x-4}{x-6}$

4. $\dfrac{a}{x-a}-\dfrac{b}{x-b}=\dfrac{a-b}{x-c}$

5. Solve for x and y: $x^2 + y^2 = 74$; $xy = 35$

(B) Factor fully:

1. $x^3y^3 - 512$

2. $16a^4 - 81b^4$

(C) Simplify:

1. $\dfrac{1}{(a-b)(a-c)}+\dfrac{1}{(b-c)(b-a)}+\dfrac{1}{(c-a)(c-b)}$

2. $\dfrac{1}{x-\dfrac{1}{x+\dfrac{1}{x}}}-\dfrac{1}{x+\dfrac{1}{x-\dfrac{1}{x}}}$

(D) Solve each of the following story problems:

1. Albertine and Boris start from the same place walking at different rates; when Albertine has walked 15 miles Boris doubles his pace, and 6 hours later passes Albertine; if Albertine walks at the rate of 5 miles an hour, what is Boris's rate at first?

2. Archy and Mehitabel received the same sum for their labor; but if one had received $10 more, and the other $8 less, then one would have had three times as much as the other. What did each receive?

3. A person selling a horse for $72 finds that her loss per cent is one-eighth of the number of dollars that she paid for the horse. What was the cost price?

4. A cistern can be filled by two pipes in 33 1/3 minutes; if the larger pipe takes 15 minutes less than the smaller to fill the cistern, find in what time it will be filled by each pipe singly.

5. Find a number whose square diminished by 119 is equal to ten times the excess of the number over 8.

6. A man is five times as old as his son, and the sum of the squares of their ages is equal to 2106; find their ages.

7. The sum of the reciprocals of two consecutive integers is 15/56. Find them.

8. The perimeter of a rectangular field is 500 yards, and its area is 14400 square yards; find the length of the sides.

9. A broker bought as many shares of stock as cost her $1875; she reserved 15, and sold the remainder for $1740, gaining $4 a share on their cost price. How many shares did she buy?

10. A prairie in the Botanic Gardens is 50 feet long and 34 feet broad. It has a path of uniform width round it; if the area of the path is 540 square feet, find its width.

* Hall & Knight, **Elementary Algebra**, revised and enlarged for use of American schools, Macmillan & Co (1896)

Available as a Google book:
https://play.google.com/store/books/details?id=SdU2AAAAMAAJ

Nothing proves more clearly that the mind seeks truth, and nothing reflects more glory upon it, than the delight it takes, sometimes in spite of itself, in the driest and thorniest researches of algebra.

- Bernard de Fontenelle, 1708