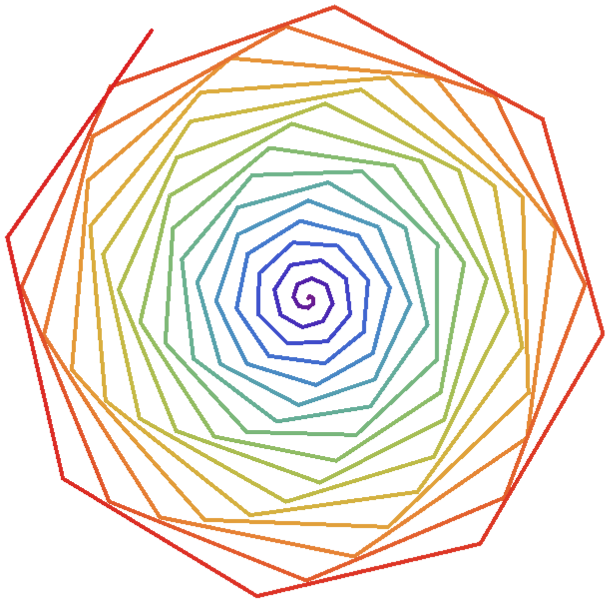
WORKSHEET X

PARAMETRIC EQUATIONS – A BRIEF INTRODUCTION



1. Sketch the curve $x(t) = 3t$, $y(t) = t^2 + 1$. Express *y* as a function of *x*.
2. Describe the parameterized curve $x(t) = 3 \cos t$, $y(t) = 4 \cos t$, $0 \le t \le 2\pi$.

   What is the relationship between the given curve above and each of the following?

   (a) $x(t) = -3 \cos t$, $y(t) = 4 \cos t$, $0 \le t \le 2\pi$.

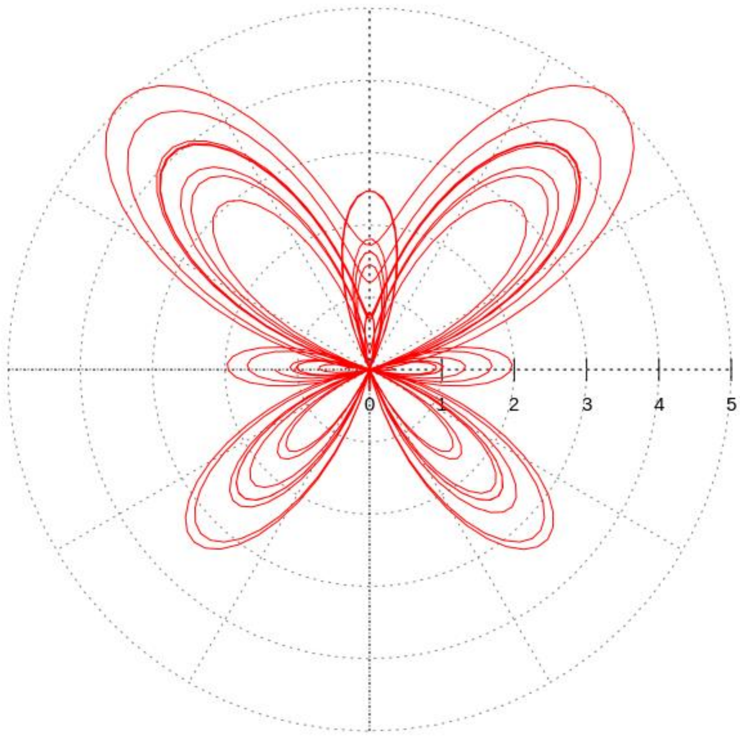
   (b) $x(t) = 3 \cos 2t$, $y(t) = 4 \cos 2t$, $0 \le t \le 2\pi$.

   (c) $x(t) = 1 - 3 \cos 2t$, $y(t) = 1 - 4 \cos 2t$, $0 \le t \le 2\pi$.
3. Show that the following is a parameterization of the cycloid:

   $$x(\theta) = a(\theta - \sin \theta),\ y(\theta) = a(1 - \cos \theta),\quad -\infty < \theta < \infty.$$
4. Show that $x = a \cos t + h$, $y = b \sin t + k$, $0 \le t \le 2\pi$, is a parametric equation of an ellipse with center at (h, k) and axes of length $2a$ and $2b$.
5. Find a parameterization of the straight line $y = 3x + 4$.
6. Find a parameterization of the straight line segment joining the points $P = (3, 5)$ to $Q = (7, 11)$.

7. Find a parameterization of the curve $y = x^2$ from $P = (-1, 1)$ to $Q = (4, 16)$.

8. Generalize problem 7 for any curve of the form $y = f(x)$ from $x = a$ to $x = b$.



the butterfly curve