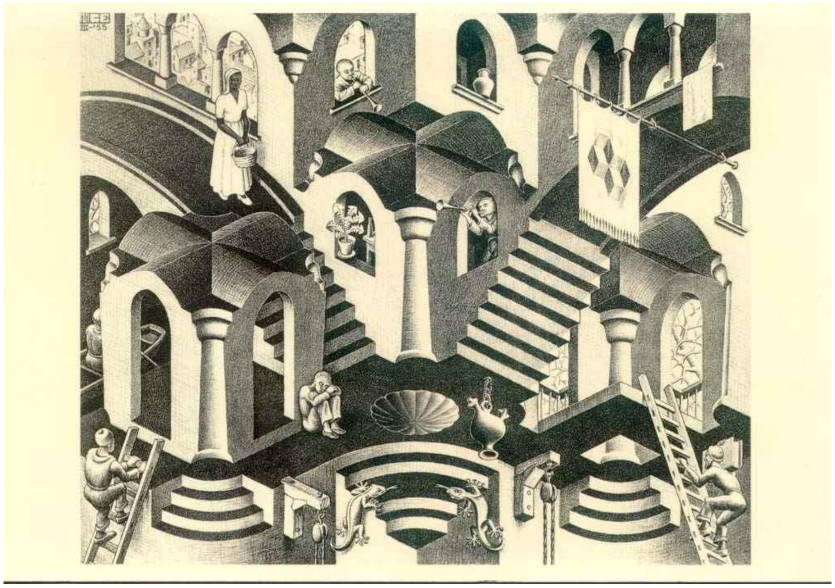
WORKSHEET XII

CURVE SKETCHING



M. C. Escher: Concave and Convex

1. Sketch each of the following curves, y = f(x). Follow the three-stage plan: (1) precalculus analysis, (2) first-derivative analysis (finding all critical points and identifying local & global extrema), and (3) second-derivative analysis.

(a) $y = 2x^3 - 14x^2 + 22x - 13$

(b) $y = x^4 - 4x^3 + 10$

(c) $y = xe^x$

(d) $y = x^4(x - 5)$

(e) $y = x^2 \ln x$

(f) $y = x\, e^{-2x}$

(g) $y = (x - 1)^4(x - 2)^9$

$(h)\quad y = \dfrac{(x+1)^2}{1+x^2}$

$(i)\quad y = \dfrac{(x-1)^2}{(x+3)^2}$

$(j)\quad y = e^{-(x-3)^2}$

$(k)\quad y = \dfrac{(x-1)^2}{(x+3)^2}$

$(l)\quad y = x^3(2x-5)^8$

(m) $y = x + \sin x$

(n) $y = x + 2 \cos x$

(0) $y = e^{2/x}$

(p) $y = (x^2 + 4)/ (2x)$

2. Determine all local and global extrema of the following functions, each defined on a *closed and bounded* interval.

(a) $y = x + 4/x$ on [1, 3]

(b) $y = \sqrt{5 - x^2}$ *on* $[-1,1]$

(c) $y = x^{10} - 10x$ *on* $[-1, 2]$

(d) $y = x^3 + 6x^2 + 1$ *on* $[-1,1]$

(e) $y = x^3 + x^5 + x^7$ *on* $[-1,1]$

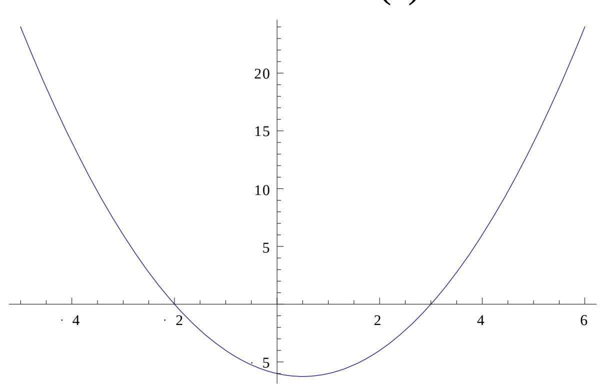
(f) $y = x \sin x$ *on* $[0, \frac{\pi}{3}]$

(g) $y = x^2 + \frac{16}{x^2}$ *on* $[-2,-1]$

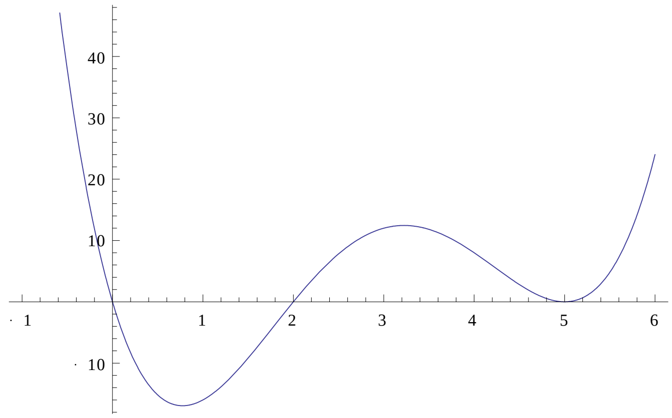
(h) $y = \frac{x}{x^4 + 48}$ *on* $[-10,10]$

3. For each graph of $y = g'(x)$ given below, draw the graphs of $y = g(x)$ and that of $y = g''(x)$.

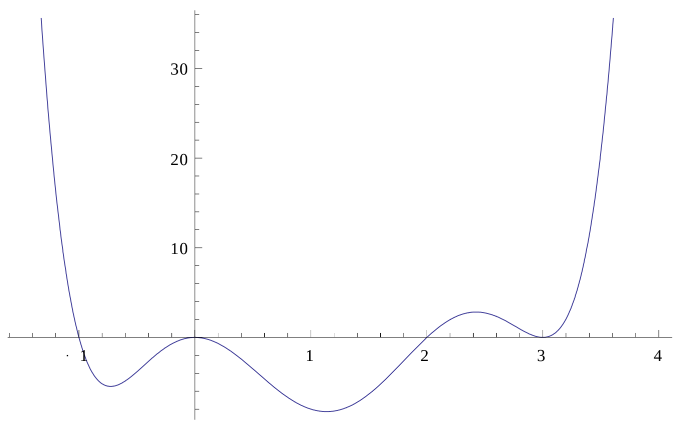
(a)



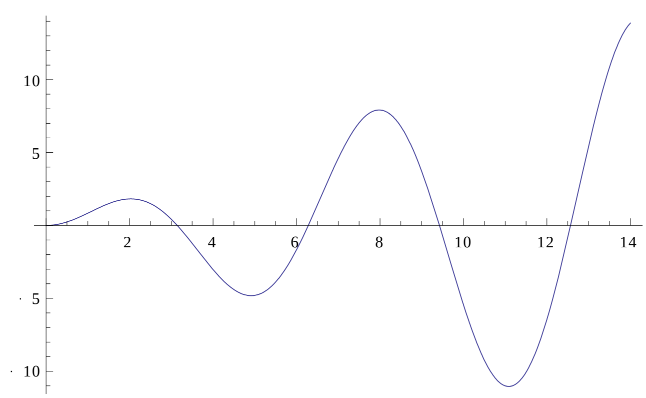
(b)



(c)



(d)



4. Given that the derivative of a smooth function $y = f(x)$ is

$$y' = (x-1)^2(x-2)(x-4)$$

Determine all points (if any) at which y has a local minimum, local maximum, or point of inflection.

5. Given that the second derivative of a smooth function $y = f(x)$ is

$$y'' = x(x-3)^2(x-2)^3(x-4)(x-9)^{2014}$$

find any and all points of inflection.

6. What is meant by the **First Derivative Test** for finding local extrema?

What is the **Second Derivative Test** for finding local extrema?

7. Use the *Second Derivative Test* to find local extrema of each of the following curves:

(a) $y = x^4 - 4x^3$

(b) $y = x^4/4 - 2x^3 + 6$

(c) $y = 3x^5 - 5x^3 + 3$

Everyone knows what a curve is, until he has studied enough mathematics to become confused through the countless number of possible exceptions.

\- Felix Klein