

# WORKSHEET XIV

## L'HÔPITAL'S RULE



Marquis Guillaume de l'Hôpital (1661 – 1704)

**I** Evaluate each of the following limits, using l'Hôpital's rule when appropriate:

$$(A) \lim_{x \rightarrow 0} \frac{e^{3x} - 1}{x}$$

$$(B) \lim_{x \rightarrow \infty} \frac{x^2}{e^x}$$

$$(C) \lim_{x \rightarrow \infty} e^{-x} \sqrt{x}$$

$$(D) \lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x$$

$$(E) \quad \lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1}{x}$$

$$(F) \quad \lim_{x \rightarrow \infty} x^{1/x}$$

$$(G) \quad \lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1 - x/2}{x^2}$$

$$(H) \quad \lim_{x \rightarrow 0} \frac{e^x - 1 - x}{x^2}$$

$$(I) \quad \lim_{x \rightarrow 0} \frac{\cos x - 1 + x^2/2}{x^4}$$

$$(J) \quad \lim_{x \rightarrow 0} \frac{x(\cosh x - 1)}{\sinh x - x}$$

$$(K) \quad \lim_{x \rightarrow \infty} \frac{\ln(ax + b)}{\ln(cx + d)} \quad (\text{where } a, b, c, d \text{ are positive constants})$$

$$(L) \quad \lim_{x \rightarrow 1} \frac{x^{2013} - 1}{x - 1}$$

$$(M) \quad \lim_{x \rightarrow \pi/2^-} \frac{\tan x}{\sec x + 1}$$

$$(N) \quad \lim_{x \rightarrow 0^+} (\sin x)^x$$

$$(O) \quad \lim_{x \rightarrow 1} \frac{\ln x}{x - 1}$$

$$(P) \quad \lim_{x \rightarrow 0^+} x \ln x$$

$$(Q) \quad \lim_{x \rightarrow \infty} \left(1 + \frac{5}{x}\right)^x$$

$$(R) \quad \lim_{x \rightarrow 0} \left(\frac{1}{\sin x} - \frac{1}{x}\right)$$

$$(S) \quad \lim_{x \rightarrow 1^+} x^{\frac{1}{x-1}}$$

$$(T) \quad \lim_{x \rightarrow \infty} \frac{\int_1^x \frac{t^4 + t + 1}{t^2 + 4t + 13 \ln t} dt}{(2x + 1)^3}$$

$$(U) \quad \lim_{x \rightarrow 1^+} \left( \frac{1}{\ln x} - \frac{1}{x-1} \right)$$

(V) Explain what happens if one tries to use l'Hopital's rule on the

following: 
$$\lim_{x \rightarrow \infty} \frac{x}{\sqrt{x^2 - 1}}$$

**II** For each of the following, find the value for  $c$  that makes the function continuous at  $x = 0$ .

$$(A) \quad f(x) = \begin{cases} \frac{4x - 2 \sin 2x}{2x^3} & \text{if } x \neq 0 \\ c & \text{if } x = 0 \end{cases}$$

$$(B) \quad g(x) = \begin{cases} (e^x + x)^{1/x} & \text{if } x \neq 0 \\ c & \text{if } x = 0 \end{cases}$$

**III** The *Gamma Function*,  $\Gamma(n)$ , is defined in terms of the integral of the function  $f(x) = x^{n-1}e^{-x}$  for  $n > 0$ . Show that for any fixed value of  $n$ , the limit of  $f(x)$  as  $x \rightarrow \infty$  is 0.

*The notion of infinity is our greatest friend; it is also the greatest enemy of our piece of mind.*

- James Pierpont