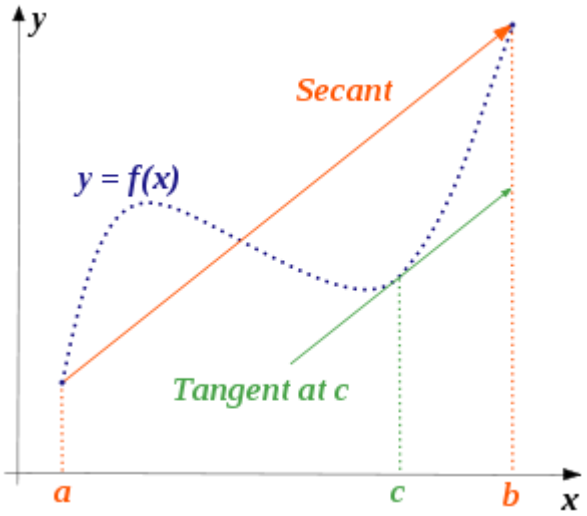
WORKSHEET XV

MVT, ANTI-DERIVATIVES, INDEFINITE INTEGRALS & INITIAL VALUE PROBLEMS



Math Bridge in Beijing

I (a) State *Rolle's Theorem*.

(b) State the *Mean Value Theorem*, and explain its geometric meaning.

(c) How is the MVT derived from Rolle's Theorem?

(d) Using the Mean Value Theorem, prove that if df/dx = dg/dx on (a, b), then there exists a constant *C* for which f(x) = g(x) + C for all $x \in (a,b)$.

(e) Let $f(x) = x^3 - 2x + 3$ be defined on the interval [1, 3]. Apply the MVT to this function and find the corresponding value of *c*.

(f) Let $g(x) = 1 + 3 \sin 2x$ be defined on the interval $[0, \pi/12]$. Apply the MVT to this function and find the corresponding value of *c*.

II Evaluate each of the following *indefinite integrals* (using the method of "judicious guessing"):

(*a*) $\int \frac{x^4 + x^3 + x + 1}{x}\, dx$

(*b*) $\int \frac{e^x}{1 + 4e^x}\, dx$

(*c*) $\int x^2 e^{4x^3}\, dx$

(*d*) $\int \frac{\sec^2 x}{1 + \tan x}\, dx$

(*e*) $\int \left(\frac{1}{x^2} + \frac{3}{x^2 + 1} \right) dx$

(*f*) $\int \ln x\, dx$ (*Try* $x \ln x$ *as a first guess.*)

(*g*) $\int \frac{\cos\left(\frac{1}{x}\right)}{x^2}\, dx$

(*h*) $\int x^2 (11x^3 + 99)^{51}\, dx$

(*i*) $\int t \sqrt[4]{1 + 2t^2}\, dt$

(*j*) $\int \frac{1}{(\arcsin z)\sqrt{1 - z^2}}\, dz$

III Solve each of the following *differential equations* (using the method of "judicious guessing").

(*a*) $\dfrac{dy}{dx} = \left(x + \dfrac{1}{x} \right)^2$

(*b*) $\dfrac{dy}{dx} = \sin^2 x \cos x$

(*c*) $\dfrac{dy}{dx} = (1 + 3\ln x)\dfrac{1}{x}$

(*d*) $\dfrac{d^2y}{dx^2} = \sinh x$

(*e*) $\dfrac{dy}{dx} = \dfrac{\pi}{4}\sec^2\left(\dfrac{\pi}{4}x\right) - \dfrac{2\ln x}{x}$

IV Solve each of the following *initial value problems* (using the method of "judicious guessing"):

(*a*) $\dfrac{dy}{dx} = 1 + x + \sin \pi x, \;\; y(0) = 5$

(*b*) $\dfrac{dy}{dx} = \tan^2 x, \;\; y(0) = 7$

(*c*) $\dfrac{dy}{dx} = \dfrac{x^2}{x^3 + 1} + x^3 + x + 7, \quad y(0) = 4$

(*d*) $\dfrac{dy}{dx} = (x + 5)\sqrt{x}, \quad y(1) = 1$

(*e*) $\dfrac{dy}{dx} = \dfrac{\sqrt{\ln x}}{x}, \quad y(e) = 11$

V Charlotte the spider is traveling along the x-axis with acceleration, a(t), given by:

$$a = \sqrt{t} - \frac{1}{\sqrt{t}}$$

Assume that at time t = 0 minute her velocity, v(0), is 4/3 cm/min and her position, x(0), is – 4/15 cm. Where is Charlotte at time t = 5 minutes?

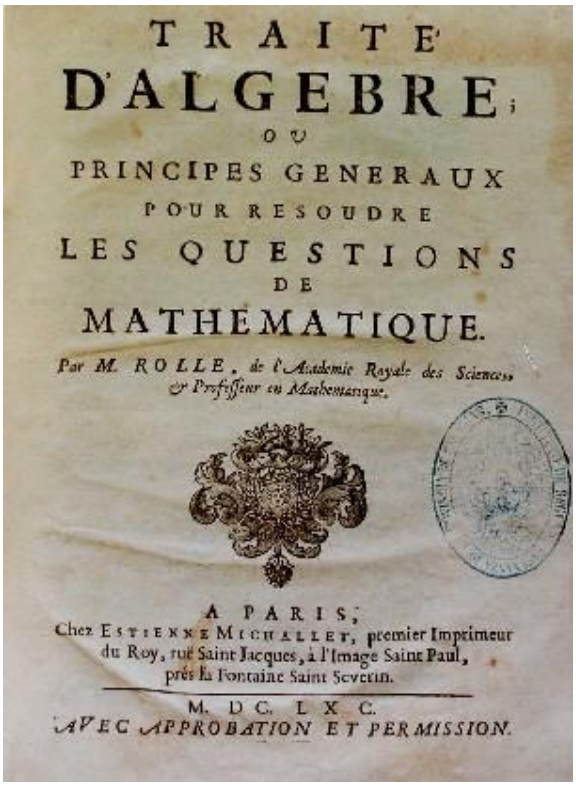
VI A grapefruit thrown upward has an initial velocity of 64 ft/sec from an initial height of 80 feet. (Recall that the acceleration due to gravity is -32 ft/sec^2.)

(a) Find the position, s(t), of the grapefruit as a function of time *t*.

(b) When does the grapefruit hit the ground?

VII Verify the following integration formula:

$$\int e^x \sin x \, dx = \frac{1}{2}\left(e^x \sin x - e^x \cos x\right) + C$$


TRAITE'
D'ALGEBRE;
OU
PRINCIPES GENERAUX
POUR RESOUDRE
LES QUESTIONS
DE
MATHEMATIQUE.
Par M. ROLLE, de l'Académie Royale des Sciences, & Professeur en Mathematique.
A PARIS,
Chez Estienne Michallet, premier Imprimeur du Roy, rue Saint Jacques, à l'Image Saint Paul, près la Fontaine Saint Severin.
M. DC. LXC.
AVEC APPROBATION ET PERMISSION.

Michel Rolle (1652 –1719)