WORKSHEET XVI

NEWTON'S METHOD

1. Using *Newton's method*, estimate the positive solution to $x^2 - 3 = 0$. Note that the Intermediate Value Theorem guarantees the existence of such a solution. (Why?)

   Start with an initial guess of $x_0 = 2$. (Of course, we know the exact answer before we begin the process, but we can better appreciate how quickly Newton's method converges to the root.)

2. Using Newton's method, estimate the solutions to the equation $x^2 + x - 1 = 0$. Start with $x_0 = -1$ for the solution on the left and $x_0 - 1$ for the solution on the right. In each case, find x_2.

3. Use Newton's method to estimate the one real solution of $x^3 + 3x + 1 = 0$. Begin with $x_0 = 0$ and then find x_2. (Explain why there is only one real root.)

4. Find a solution to the equation $x = 1 + 0.5 \sin x$ using Newton's method. Graphing would suggest that there is a solution near $x = 1.5$.

5. Let $G(x) = x^4 - 3x^3 + 4x - 1$. Walt wants to find a root of the equation $G(x) = 0$. First he observes that $G(0) < 0$ and $G(1) > 0$. Then letting $x_0 = 0.5$, Walt employs Newton's method to find better approximations to the root between $x = 0$ and $x = 1$. Find x_1 and x_2 (each to 5 significant digits). Show your work!

When I am working on a problem I never think about beauty. I only think about how to solve the problem. But when I have finished, if the solution is not beautiful, I know it is wrong.

- Buckminster Fuller (1895-1983)