WORKSHEET XVII (REVISED)

RIEMANN SUMS

Bernhard Riemann (1826 –1866)

1. Charlotte, the spider, lives on the x-axis. Suppose that, at time t = 1 minute, she is at x = 5 cm, and that her velocity (in cm/minute) at time *t* is given by:

$$v(t) = 4t + 1.$$

Where is Charlotte at time *t = 4 minutes*?

2. For each of the following functions and associated partitions, compute the left-endpoint sum, the right-endpoint sum, and the midpoint sum:

(a) $f(x) = 2x + 5$; $P = \{-2, -1, 0, 1\}$

(b) $f(x) = 2x^2 + 1$; $P = \{-1, 0, 1, 2\}$

(c) $f(x) = 1/x^2$; $P = \{1, 3/2, 2, 5/2, 3\}$

(d) $f(x) = \sin x$; $P = \{0, \pi/4, \pi/2, 3\pi/4, \pi\}$

(e) $f(x) = x^3$; $P = \{1, 2, 3, 4\}$

3. Compute each of the following sums:

$$(a) \quad \sum_{n=1}^{4} n^2$$

$$(b) \quad \sum_{k=0}^{5} k(k-1)$$

(c) $\sum_{j=-3}^{1} 1/(j+4)$

4. Find a formula for

(a) $\sum_{k=1}^{n} k$

(b) $\sum_{j=1}^{n} j^2$

5. Using Archimedes' *Method of Exhaustion*, find the area under the parabola $f(x) = x^2$ that lies above the interval [0, 1] on the x-axis.

6. How would you define the *average value* of a continuous function f(x) on an interval [a, b]? Using your definition, find (or estimate) the *average value* of each of the following functions:

(a) $f(x) = x^2$ over [0, 1]

(b) $g(x) = 3x + 1$ over [7, 11]

(c) $h(x) = |x - 2|$ over [0, 3]

(d) $F(x) = \sin x$ over $[0, \pi]$

7. Suppose that $\sum_{n=0}^{100} a_n = 13$ and that $\sum_{n=0}^{100} b_n = 5$. Find:

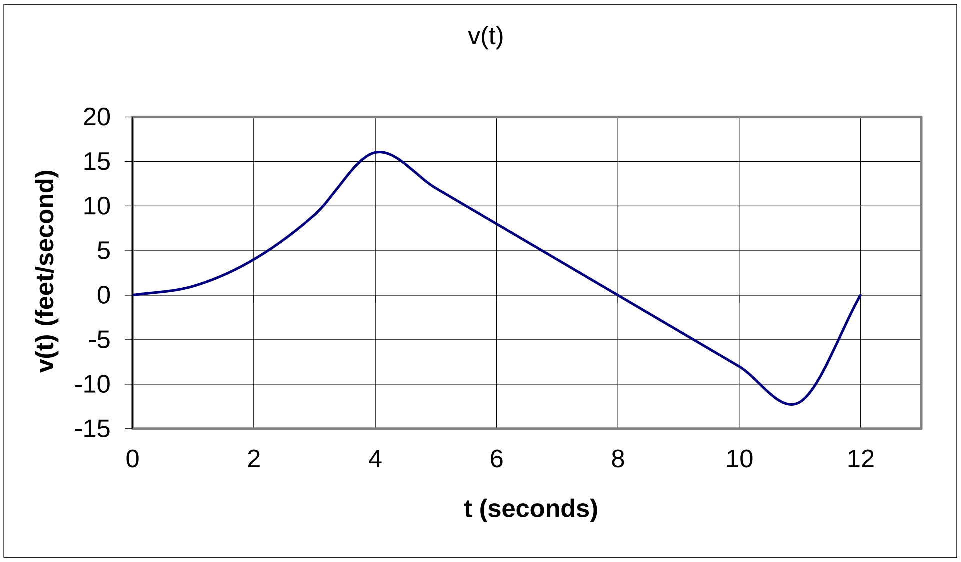
(a) $\sum_{n=0}^{100} 3a_n$

(b) $\sum_{n=0}^{100} (a_n - 1)$

(c) $\sum_{n=0}^{100} (a_n - b_n)$

(d) $\sum_{n=0}^{100} (5a_n - 3b_n + 3)$

8. Albertine launches a model rocket from the ground at time t = 0. The rocket starts by traveling straight up in the air. The graph below illustrates the upward velocity of the rocket as a function of time.



(a) Sketch a graph of the *acceleration* of the rocket as a function of time.

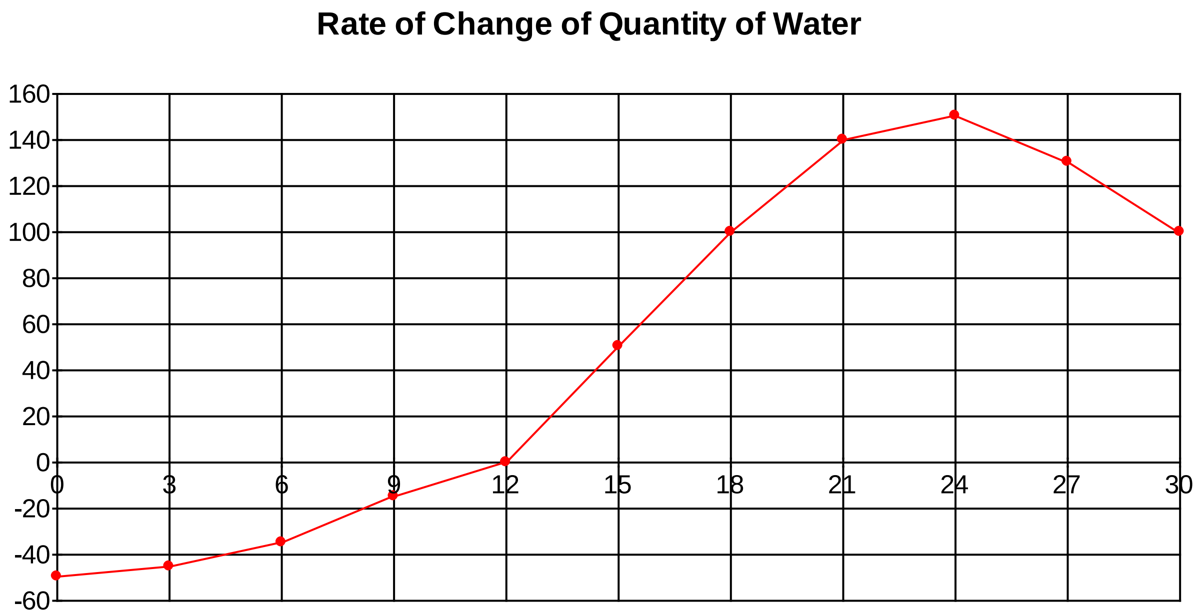
(b) Sketch a graph of the *height* of the rocket as a function of time.

(c) Give an estimate of the *maximum height* the rocket achieved.

9. Swann's old rowboat has sprung a leak. Water is flowing into the boat at a rate, *r(t)*, given in the following table. Compute upper and lower estimates for the volume of water that has flowed into Swann's boat during the 15 minutes. Draw a graph to illustrate the lower estimate.

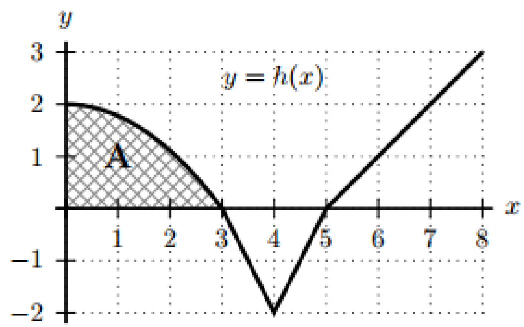
t minutes	0	5	10	15
r(t) liters/min	12	20	24	16

10. The graph below shows the *RATE OF CHANGE* of the quantity of water in the Water Tower of OZ, in liters per day, during the month of April, 2015. The tower contained 12,000 liters of water on April 1. *Estimate* the quantity of water in the tower on April 30.   Show your work.



1. [13 points]

The graph of a function $h(x)$ is shown on the right. The area of the shaded region A is 4, and $h(x)$ is piecewise linear for $3 \leq x \leq 6$.



Compute each of the following. If there is not enough information to compute a value exactly, write NOT ENOUGH INFO.

a. [2 points] Find $\int_0^3 (h(x)+2)\, dx$.

Answer: $\int_0^3 (h(x)+2)\, dx =$ ____________________

b. [2 points] Find the average value of $h(x)$ on the interval $[0, 4]$.

Answer: ____________________

c. [3 points] Let $J(x) = \sin(\pi h(x))$. Find $J'(3.5)$.

Answer: $J'(3.5) =$ ____________________

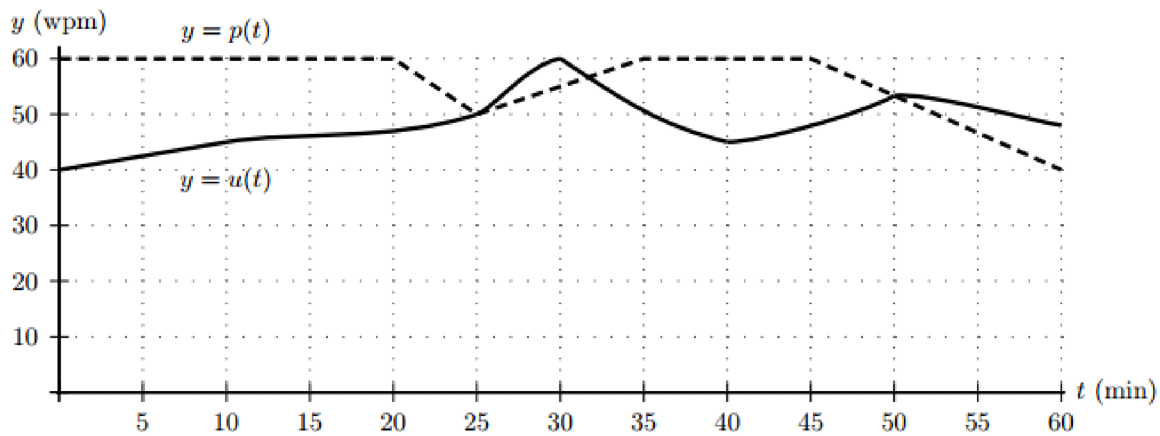
d. [3 points] Let $H(x)$ be an antiderivative of $h(x)$ with $H(4) = 5$. Find an equation for the tangent line to the graph of $H(x)$ at $x = 4$.

Answer: ____________________

e. [3 points] Let $g(x) = e^x$. Find $\int_6^7 \left(g(x)h'(x) + g'(x)h(x)\right)\, dx$.

Answer: $\int_6^7 \left(g(x)h'(x) + g'(x)h(x)\right)\, dx =$ ____________________

7. [10 points] A history professor gives a 60 minute lecture, while one eager undergraduate student takes notes by typing what the professor says, word for word. Unfortunately, the student cannot always type as quickly as the professor is speaking.
Functions p and u are defined as follows. When t minutes have passed since the start of the lecture, the professor is speaking at a rate of $p(t)$ words per minute (wpm) while the undergraduate student is typing at a rate of $u(t)$ words per minute (wpm). Shown below are graphs of $y = p(t)$ (dashed) and $y = u(t)$ (solid).



a. [2 points] How many minutes after the start of the lecture is the student typing most quickly?

Answer: ____________________

b. [3 points] Write a definite integral equal to the number of words the student types between the start of the lecture and the time the professor reaches the 600th **word** of the lecture. You do not need to evaluate the integral.

Answer: ____________________

c. [3 points] How many minutes after the start of the lecture is the student furthest behind in typing up the lecture? (In other words, after how many minutes is the difference between the total number of words the professor has spoken and the total number of words the student has typed the greatest?)

Answer: ____________________

d. [2 points] What is the average rate, in words per minute, at which the professor is speaking between $t = 40$ and $t = 60$?

Answer: ____________________

I do hate sums. There is no greater mistake than to call arithmetic an exact science. There are hidden laws of number which it requires a mind like mine to *perceive. For instance, if you add a sum from the bottom up, and then again from the top down, the result is always different.*

- Mrs. La Touche

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