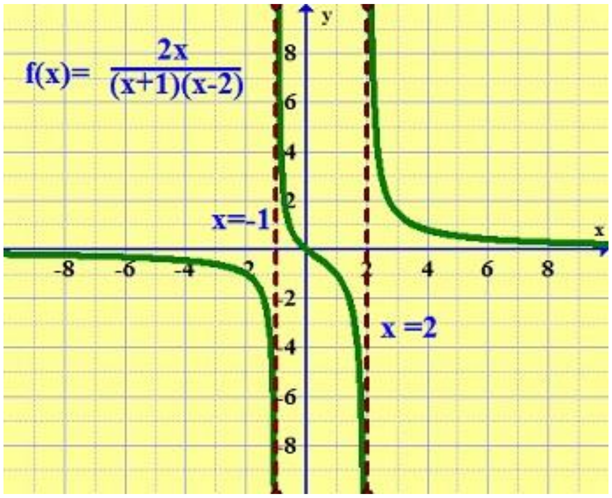
WORKSHEET II

More about Functions: graphing rational functions; introduction to hyperbolic functions



1. *Sketch the graph* of each of the following rational functions. This includes locating zeroes, locating singularities, doing a sign analysis, and studying limiting behavior.

(A) $y = x^3(x-1)^4(x-2)^5$

(B) $y = \dfrac{x^2}{(x-3)(x-5)}$

(C) $y = \dfrac{x^2(x+3)}{x-7}$

(D) $y = \dfrac{x(x-2)(x+3)}{(x+1)(x-1)(x-5)}$

2. Suppose that $f(x) \to \infty$ and $g(x) \to \infty$ as $x \to \infty$. We say that *g goes to infinity faster than f* if $\dfrac{f(x)}{g(x)} \to 0$ as $x \to \infty$. Also, we say that "*f and g go to infinity at roughly the same rate*" if $\dfrac{f(x)}{g(x)} \to L$ as $x \to \infty$ where $0 < L < \infty$.

For each of the following pairs of functions determine if one goes to infinity faster than the other or if they go to infinity at roughly the same rate.

(A) $y = 3x^2 + 11$, $y = x^5 + x + 99$

(B) $y = 2^x$, $y = x^{100}$

(C) $y = 3^x$, $y = e^x$

(D) $y = \ln x$, $y = x$

(E) $y = \sqrt{x}$, $y = \sqrt[3]{x}$

(F) $y = \ln x$, $y = \sqrt{x}$

(G) $y = (x^2+1)^4$, $y = (2x+1)^3x^5$

(H) $y = 4x$, $y = \sqrt{x^2 + 9}$

(I) $y = \ln x$, $y = \cos x + \ln x$

(J) $y = \ln x$, $y = \ln(\ln x)$

3. Define the *hyperbolic functions* sinh x, cosh x, tanh x and sech x. Sketch each of these functions.

(A) Prove the identity: $\cosh^2 x - \sinh^2 x = 1$

(B) Prove the identity: $1 - \tanh^2 x = \text{sech}^2 x$

Jasper Johns: CATENARY (JACOB'S LADDER) – 1999

"...he seemed to approach the grave as a hyperbolic curve approaches a line, less directly as he got nearer, till it was doubtful if he would ever reach it at all."

- Thomas Hardy, ***Far from the Madding Crowd***

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