WORKSHEET XX

AREA

1. Find the area of the region enclosed by the parabola $y = 2 - x^2$ and the line $y = -x$.
2. Find the area of the region in the first quadrant bounded above by $y = x^{1/2}$ and below by the x-axis and the line $y = x - 2$.
3. Repeat exercise (2) above, but this time integrate with respect to *y*.
4. Find the area of the crescent-shaped region in the first quadrant that is bounded by $y = x^{13}$ and $y = x^{15}$.
5. Find the area of the region bounded by $y = 7 - 2x^2$ and $y = x^2 + 4$.
6. Find the area of the region enclosed by $y = x^4 - 4x^2 + 4$ and $y = x^2$.
7. Find the area of the region enclosed by $y = x^4 - 4x^2 + 4$ and $y = x^2$.
8. Find the area of the region enclosed by $y = x(a^2 - x^2)^{1/2}$, where $a > 0$, and $y = 0$.
9. Find the area of the region enclosed by $y = (|x|)^{1/2}$ and $5y = x + 6$.
10. Find the area of the region enclosed by $x = y^3 - y^2$ and $x = 2y$.
11. Find the area of the region bounded by $4x^2 + y = 4$ and $x^4 - y = 1$.
12. Find the area of the region enclosed by $y = 2 \sin x$ and $y = \sin (2x)$, $0 \le x \le \pi$.
13. Find the area of the region enclosed by $y = \cos(\pi x/2)$ and $y = 1 - x^2$.
14. Find the area of the region enclosed by $y = \sin(\pi x/2)$ and $y = x$.
15. Find the area of the "triangular" region in the first quadrant that is bounded above by the curve $y = e^{2x}$, below by the curve $y = e^x$, and on the right by the line $x = \ln 3$.

DIFFERENTIATING INTEGRALS

Differentiate with respect to x each of the following integrals using the FTC and Leibniz's Formula:

1. $y = \int_3^x \sqrt{5 + \cos^3 t}\, dt$

2. $y=\int_{1}^{x}\frac{5}{3+t^{4}}\,dt$

3. $y=\int_{\sec x}^{4}\frac{1}{1+t^{2}}\,dt$

4. $y=\int_{1/x}^{x}\frac{1}{t}\,dt$

5. $y=\int_{\cos x}^{\sin x}\frac{1}{1-t^{2}}\,dt$

6. $y=\int_{\sqrt{x}}^{x^{2}}\frac{e^{t}}{t}\,dt$

USING INTEGRALS TO APPROXIMATE RIEMANN SUMS

Evaluate each of the following limits:

1. $\lim_{n\to\infty}\frac{1^{5}+2^{5}+3^{5}+\ldots+n^{5}}{n^{6}}$

2. $\lim_{n\to\infty}\frac{1^{3}+2^{3}+3^{3}+\ldots+n^{3}}{n^{4}}$

3. $\lim_{n\to\infty}\frac{1}{n}\left(\sin\frac{\pi}{n}+\sin\frac{2\pi}{n}+\sin\frac{3\pi}{n}+\ldots+\sin\frac{n\pi}{n}\right)$

The nicest child I ever knew
Was Charles Augustus Fortescue.
He never lost his cap, or tore
His stockings or his pinafore:
In eating Bread he made no Crumbs,
He was extremely fond of sums.

- Hilaire Belloc, **Cautionary Tales** (1907)