

WORKSHEET XXI

SUBSTITUTION IN INDEFINITE & DEFINITE INTEGRALS

I Use the *method of substitution* to evaluate each of the following indefinite integrals.

$$(A) \int x\sqrt{2x-1} dx$$

$$(B) \int x(x+1)^{\frac{1}{3}} dx$$

$$(C) \int x^2(x+1)^{2015} dx$$

$$(D) \int \tan x dx = \int \frac{\sin x}{\cos x} dx$$

$$(E) \int \sec x dx = \int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x} dx$$

$$(F) \int \tan(4x+1)\sec^2(4x+1) dx$$

$$(G) \int \frac{x}{\sqrt{1-x^4}} dx$$

$$(H) \int \frac{e^x}{1+e^{2x}} dx$$

$$(I) \int \frac{\sin z}{(3+2\cos z)^2} dz$$

$$(J) \int \frac{t^2-5}{t+2} dt$$

$$(K) \int \frac{\sqrt{t}}{\left(1+t^{\frac{3}{2}}\right)^2} dt$$

$$(L) \int \frac{\sqrt{\ln x}}{x} dx$$

$$(M) \int x^7(5+x^8)^5 dx$$

$$(N) \int \frac{\sec^2 y}{1 - \tan y} dy$$

$$(O) \int (\tan^4 u)(\sec^2 u) du$$

$$(P) \int \sqrt{a + bx} dx$$

$$(Q) \int \sqrt{3 + \sqrt{x}} dx$$

$$(R) \int \frac{1}{\sqrt{1 + \sqrt{x}}} dx$$

II Integrating squares of sine and cosine.

$$(A) \sin^2 x = \frac{1 - \cos(2x)}{2}$$

$$(B) \cos^2 x = \frac{1 + \cos(2x)}{2}$$

III For each of the following integrals, use the *change of variable theorem* for definite integrals:

$$(A) \int_0^1 x(x^2 + 1)^3 dx$$

$$(B) \int_0^{\pi/10} \sin^2(5x) dx$$

$$(C) \int_0^3 \sqrt{x+5} dx$$

$$(D) \int_0^4 \sqrt{1+\sqrt{x}} dx$$

$$(E) \int_1^5 \frac{x}{\sqrt{2x-1}} dx$$

IV Let $f(x)$ be continuous on the real line. Verify that

$$(A) \int_0^1 x^a (1-x)^b dx = \int_0^1 x^b (1-x)^a dx$$

$$(B) \int_a^b f(x+h) dx = \int_{a+h}^{b+h} f(x) dx$$

$$(C) \int_{ca}^{cb} f(x) dx = c \int_a^b f(cx) dx$$

A substitute shines brightly as a king,

Until a king be by.

- Shakespeare, **The Merchant of Venice**