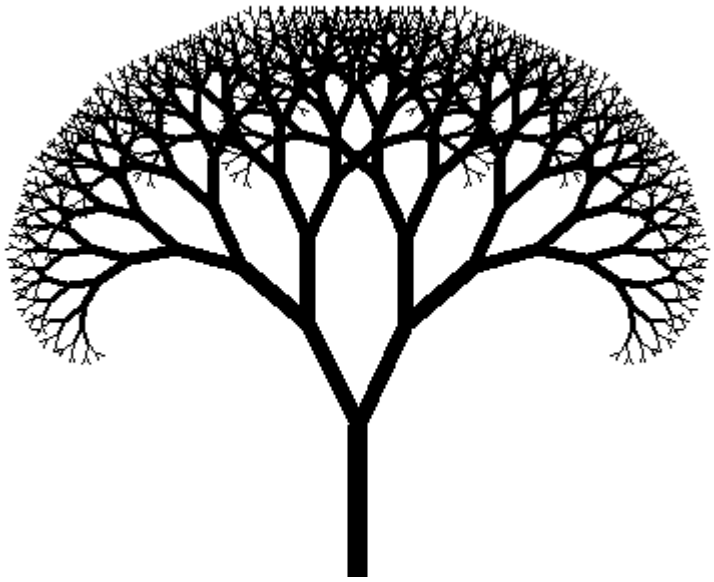
WORKSHEET III

limits & continuity



Fractal tree

I Evaluate each of the following limits or explain why the limit fails to exist.

1. $\lim\limits_{x\to 3} \dfrac{x-3}{x^2-5x+6}$

2. $\lim\limits_{x\to 3} \dfrac{x^3-27}{x-3}$

3. $\lim\limits_{x\to 1} \dfrac{x^4-1}{x^2-1}$

4. $\lim\limits_{x\to 1} \dfrac{x+9}{x^2-4}$

5. $\lim\limits_{x\to 1} \dfrac{x^4-1}{x^3-1}$

6. $\lim\limits_{x\to 0} \dfrac{|x|}{x}$

7. $\lim_{x\to 16} \dfrac{\sqrt{x}-4}{x-16}$

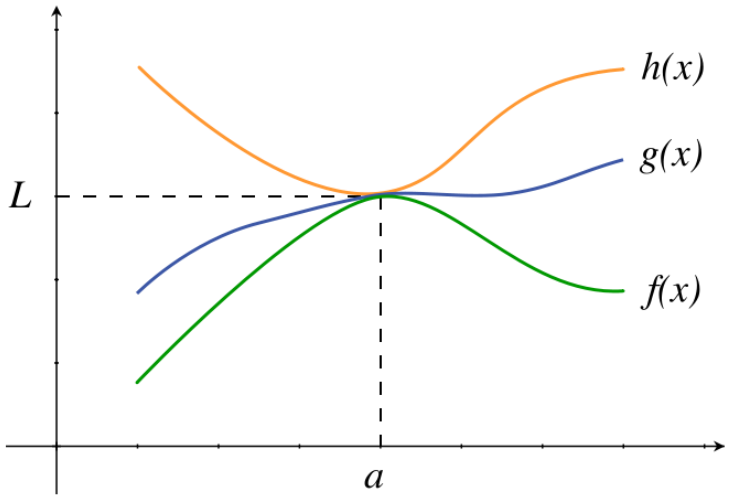
8. $\lim_{x\to 1} \dfrac{\frac{1}{x}-1}{x-1}$

9. $\lim_{x\to 4} \sqrt{\dfrac{x+5}{x+12}}$

10. $\lim_{x\to 1} \dfrac{x^2-1}{(x-1)^3}$

II State the *limit laws*.

III State the *Sandwich Theorem (*a.k.a. *Squeeze Theorem, Pinching Theorem, Two Gendarmes Theorem, Two Policemen and a Drunk Theorem).*



IV (A) Is the function f(x) = (sin x)/x *even* or *odd* or neither?

(B) Using the Sandwich Theorem prove that

(sin x)/x → 1 as x → 0.

V Define *continuity* of a function y = f(x) at x = a. What does it mean for a function to be *continuous*?

VI Consider each of the following functions and the given point on the x-axis. Does the function have a *continuous extension* at the given point? Explain.

1. $f(x)=\dfrac{x-2}{x-3},\ x=3$

2. $G(x)=\dfrac{x^2-9}{x-3},\ x=3$

3. $H(x)=\dfrac{2x^2-13x+20}{3x^2-13x+4},\ x=4$

4. $g(x)=\dfrac{2x^2-13x+20}{3x^2-13x+4},\ x=1/3$

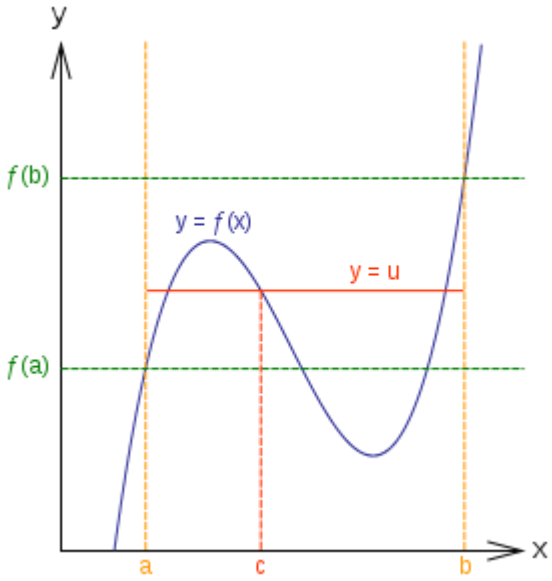
VII For which value of *a* is the following function *continuous everywhere?*

$$f(x)=\begin{cases} x^2-1 & for\ \ x<3 \\ 2ax & for\ \ x\geq 3 \end{cases}$$

VIII For which values of *a* and *b* is the following function *continuous everywhere?*

$$g(x)=\begin{cases} ax+2b & for\ \ x\leq 0 \\ x^2+3a-b & for\ \ 0<x\leq 2 \\ 3x-5 & for\ \ x>2 \end{cases}$$

IX State the *Intermediate Value Theorem*. Using the IVT, prove that the polynomial $f(x) = x^4 + 4x^3 - 20x + 11$ must have a root between $x = 0$ and $x = 1$.



X Give examples of each of the following types of discontinuities: *removable, jump, infinite, and essential*.

For each of the following functions, determine the type of discontinuity at the given point.

(a) $y = (\sin x) / x$ at $x = 0$

(b) $y = (x^3 - 8) / (x - 2)$ at $x = 2$

(c) $y = \sin (1/x)$ at $x = 0$

(d) $y = |x - 3| / (x - 3)$ at $x = 3$.

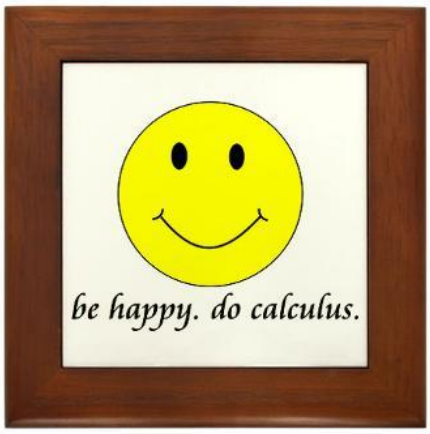
(e) $y = x \sin(1/x)$ at $x = 0$

(f) $y = (\cos x) / x$ at $x = 0$

(g) $y = (x^7 - 1)/ (x - 1)$ at $x = 1$

(h) $y = (\cosh x) / x$ at $x = 0$

(i) $y = (1 - \cos x) / x$ at $x = 0$.



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