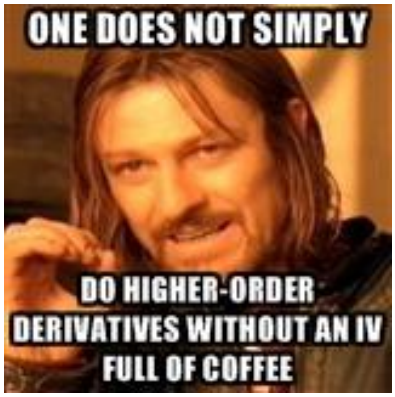
WORKSHEET VIII

HIGHER-ORDER DERIVATIVES



1. Find the first *three* derivatives of each of the following functions.

   (*A*) $y = ax^2 + bx + c$

   (*B*) $y = 2x^3 + \dfrac{1}{x^2} + e^x$

   (*C*) $y = xe^x$

   (*D*) $y = \sin x$

   (*E*) $y = x \sin x$

   (*F*) $y = x^2 \ln x$

2. (A) If $(d/dx)e^{4x} = 4e^{4x}$, find $(d^{199}/dx^{199})\, e^{4x}$.

   (B) If $(d/dx) \sin 5x = 5 \cos 5x$, and $(d/dx) \cos 5x = -5 \sin 5x$, find $(d^{2015}/dx^{2015}) \sin 5x$.

3. If $f(x) = x^{1/2}$, find $f^{(4)}(x)$.

4. If $x(t) = 3t^3 - 4t + 1$ is the position (measured in meters) of Charlotte on the x-axis at time t (measured in hours), find Charlotte's *velocity* and *acceleration* at time t = 2 hrs.

5. If $F(x) = x^m$, find $F^{(m)}(x)$. (Assume that m is a positive integer.)

6. Let y = ln x. Given that dy/dx = 1/x, find d^4y/dx^4. Can you find $d^{10}y/dx^{10}$?

7. (University of Michigan) Consider the following table giving values, rounded to three decimal places, of a function $f(x)$.

x	0	0.5	1
$f(x)$	0	0.247	0.841

a. Estimate $f'(1)$. Be sure it is clear how you obtain your answer.

b. Estimate $f''(1)$. Again, be sure that it is clear how you obtain your answer.

c. Estimate $f(1.25)$, being sure your work is clear.

d. Based on your work in (a) and (b), is your estimate in (c) an over- or underestimate? Explain.

8. (University of Michigan) A paperback book (definitely not a valuable calculus textbook, of course) is dropped from the top of Mertz hall (which is 40 m high) towards a very large, upward pointing fan. The average velocity of the book between time $t = 0$ and later times is shown in the table of data below (in which t is in seconds and the velocities are in m/s).

between $t = 0$ seconds and $t =$	1	2	3	4	5
average velocity is	−5	−10	−11.67	−9	−7.2

a. Fill in the following table of values for the height $h(t)$ of the book (measured in meters). Show how you obtain your values.

t	0	1	2	3	4	5
$h(t)$	40	_____	_____	_____	_____	_____

b. Based on your work from (a), is $h''(1) > 0$, < 0, or $= 0$? Is $h''(3) > 0$, < 0, or $= 0$? Explain.

9. For each of the descriptions of a function f that follow, indicate which of the graphs below match the description. For each description there may be no, one, or several graphs that match; write **none** if no graphs match the description. You may need to use a graph more than once. In each case you should assume that f is defined only on the domain $[0, 2]$.

- $f'(x) < 0$ for $x < 1$ and $f'(x) > 0$ for $x > 1$; $f(x) < 0$ for $x < 1$ and $f(x) > 0$ for $x > 1$; and $f(x)$ is continuous everywhere except at $x = 1$.

  matching graph(s): _____

- $f'(x) > 0$ for all $x = 16$; $f(x) < 0$ for all $x = 16$; and $f(x)$ is differentiable everywhere except at $x = 1$.

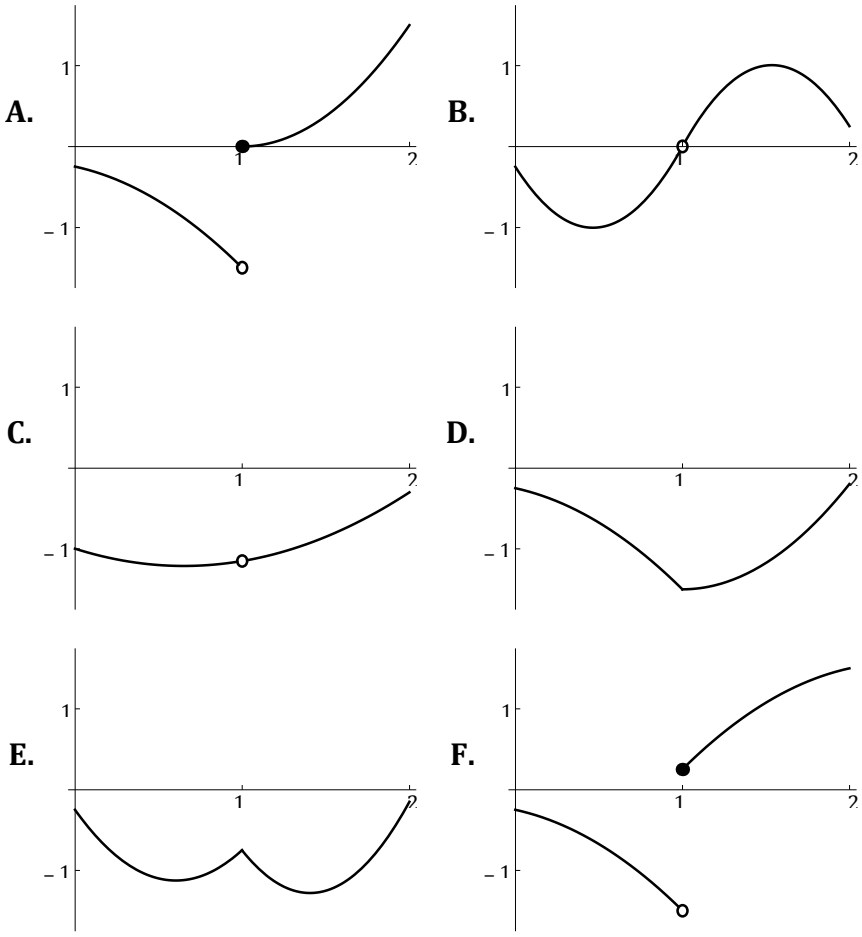
  matching graph(s): _____

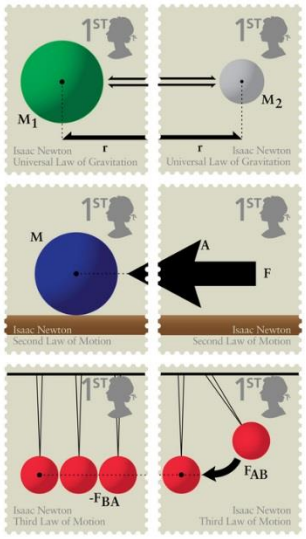
- $f'(x) < 0$ for all $x = 16$; $f(x) < 0$ for $x < 1$ and $f(x) > 0$ for $x > 1$; and $f(x) < 0$ for all $x = 1.6$

  matching graph(s): _____

- $f'(x) < 0$ for $x < 1$ and $f'(x) > 0$ for $x > 1$; $f(x) < 0$ for $x < 1$ and $f(x) > 0$ for $x > 1$; and $f(x)$ is differentiable everywhere except at $x = 1$.

  matching graph(s): _____


A.
B.
C.
D.
E.
F.
1
2
– 1


1ST
M1
M2
r
Isaac Newton
Universal Law of Gravitation
M
A
F
Second Law of Motion
-FBA
FAB
Third Law of Motion