WORKSHEET IX

Chain Rule, Implicit Differentiation, Logarithmic Differentiation, Inverse Trig functions

I Compute dy/dx using the Chain Rule:

1. $y = (1 + \sin x)^8$

2. $y = \sqrt{5 + x^3 + 2x^5}$

3. $y = e^{1+\cos x}$

4. $y = \dfrac{e^x - e^{-x}}{e^x + e^{-x}} = \tanh x$

5. $y = \sin(13\cos x)$

6. $y = e^{4x}\tan 5x$

7. $y = \sin^4 x + \sqrt{3x + 11}$

8. $y = (x+1)^5(3x-13)^7$

9. $y = \dfrac{\sec 3x}{\sqrt{2x+1}}$

10. $y = \sec(x + \ln x)$

II For each of the following curves, find all *critical points* (i.e., points for which dy/dx = 0).

1. $y = (x+1)^5(2x-1)^8$

2. $y = e^{-3x}(x+4)^9$

3. $y = \dfrac{(3x-5)^5}{(2x+1)^3}$

4. $y = x + \sin x$

5. $y = 13x + 3\sin 4x$

III 1. Given y = $\tan^2(\pi u/8)$ and u = $1 + 2x^2 - 4x^3 + 3$, find dy/dx when x=1.

2. Sketch the curve y = $(2x-1)^4(3x+1)^5$ and locate all zeroes, perform a sign analysis, study limiting behavior and locate all critical points.

3. Sketch the curve $y = e^x(x - 1)^4$ and locate all zeroes, perform a sign analysis, study limiting behavior and locate all critical points.

4. Show that the derivative of ln x is 1/x. (*Hint:* Let $y = \ln x$; then $x = e^y$.)

5. Find dy/dx if $y = \ln(\sec x + \tan x)$ and simplify your answer.

6. Find dx/dt if $x(t) = \ln(\ln(t))$.

IV Using implicit differentiation, find dy/dx:

1. $y + x = xy + 7$

2. $y^2 = x^2 + \sin xy$

3. $y \sin \frac{1}{y} = 1 - xy$

V 1. Prove the power rule for *rational* exponents:

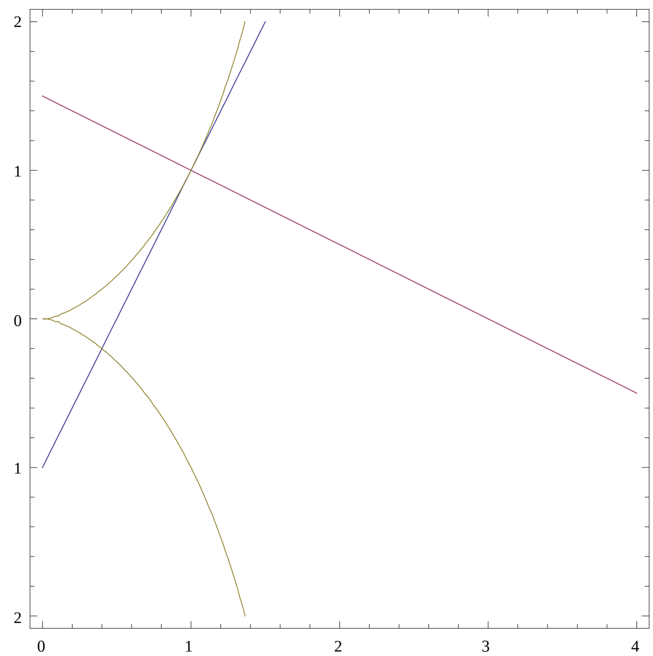
$$(d/dx)\, x^p = px^{p-1} \text{ if } p \text{ is rational.}$$

2. Find d^2y/dx^2 if $y^2 + xy = 1$.

3. Consider the curve defined implicitly by: $x^2 + xy - y^2 = 1$. Verify that the point P = (2, 3) lies on this curve. Find the equations of the *tangent* and *normal* lines to this curve at the point *P*.

4. Find equations for the *tangent* and *normal* lines to the *cissoid of Diocles* (from 200 B.C.):

$$y^2(2 - x) = x^3 \text{ at } Q = (1, 1).$$



VI Using implicit differentiation, find dy/dx for

$$y = \arcsin x,\ y = \arctan x, \text{ and } y = \text{arcsec } x.$$

VII Find dy/dx for each of the following:

1. $y = \arcsin(2x + 5)$

2. $y = \arctan\left(\frac{1}{x}\right)$

3. $y = \ln(arc\sec x)$

4. $y = (\arcsin(x^2))^5$

VIII Using logarithmic differentiation, find dy/dx for each of the following:

1. $y = x(x+1)^5(3x-4)^{11}$

2. $y = \dfrac{5x+7}{\sqrt{3x+5}}$

3. $y = \sqrt{\dfrac{x(3x+1)(2x+5)}{(x-4)(7x-1)}}$

It is often better to be in chains than to be free.

- Franz Kafka, **The Trial**