

# Math 115 — Second Midterm

November 15, 2011

Name: \_\_\_\_\_

Instructor: \_\_\_\_\_ Section: \_\_\_\_\_

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1. **Do not open this exam until you are told to do so.**
2. This exam has 9 pages including this cover. There are 9 problems. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
3. Do not separate the pages of this exam. If they do become separated, write your name on every page and point this out to your instructor when you hand in the exam.
4. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer questions about exam problems during the exam.
5. Show an appropriate amount of work (including appropriate explanation) for each problem, so that graders can see not only your answer but how you obtained it. Include units in your answer where that is appropriate.
6. You may use any calculator except a TI-92 (or other calculator with a full alphanumeric keypad). However, you must show work for any calculation which we have learned how to do in this course. You are also allowed two sides of a  $3'' \times 5''$  note card.
7. If you use graphs or tables to find an answer, be sure to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
8. **Turn off all cell phones and pagers**, and remove all headphones.
9. You must use the methods learned in this course to solve all problems.
10. You must clearly indicate your final answer for each problem to receive credit.

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Problem	Points	Score
1	9	
2	8	
3	12	
4	10	
5	8	
6	13	
7	16	
8	12	
9	12	
Total	100	

1. [9 points] Let  $U = f(t)$  give the number of Facebook users in millions in year  $t$ . Suppose  $f(2005) = 5.5$  and  $f'(2005) = 4.9$ . For this problem assume that  $f(t)$  is strictly increasing.
- a. [4 points] Find and interpret, in practical terms,  $f^{-1}(5.5)$ .

$$f^{-1}(5.5) = \underline{\hspace{10em}}$$

- b. [5 points] Showing work, evaluate  $(f^{-1})'(5.5)$ . Interpret your answer in practical terms.

$$(f^{-1})'(5.5) = \underline{\hspace{10em}}$$

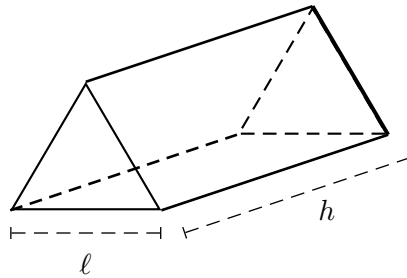
2. [8 points] Recall the function  $T(x)$  that took the number of followers (in millions) of a Twitter user and returned a value from 0 to 10 called the user's Twitter celebrity index. The derivative of  $T(x)$  is given by the function

$$T'(x) = \frac{1532.5 \cdot (0.6)^x}{(5 + 60(0.6)^x)^2}.$$

- a. [4 points] If  $T(3) = 1.56$ , compute the local linearization of  $T(x)$  near  $x = 3$ .

- b. [4 points] Use your expression from (a) to approximate the Twitter celebrity index of a celebrity with 3.2 million followers.

3. [12 points] Consider the prism with equilateral triangles of side length  $\ell$  centimeters for ends and a length of  $h$  centimeters, illustrated below. The volume of this prism is  $\sqrt{3} \ell^2 h / 4$ . You may find it useful to note that the area of an equilateral triangle of side length  $\ell$  is  $\sqrt{3} \ell^2 / 4$ .



- a. [4 points] Give the equation of the surface area of this prism, listing units.

Surface area=\_\_\_\_\_

- b. [8 points] If the prism has a fixed volume of  $16 \text{ cm}^3$ , find the values of  $\ell$  and  $h$  which minimize the surface area. Clearly justify that you have found the minimum.

4. [10 points] The cable of a suspension bridge with two supports  $2L$  meters apart hangs  $H$  meters above the ground. The height  $H$  is given in terms of the distance in meters from the first support  $x$  (in meters) by the function

$$H(x) = e^{x-L} + e^{L-x} + H_0 - 2$$

where  $H_0$  and  $L$  are positive constants. Notice that  $x$  ranges from 0 (the first support) to  $2L$  (the second support).

- a. [4 points] Find (but do not classify) the critical points for the function  $H(x)$ .

- b. [6 points] Find the  $x$  and  $y$  coordinates of all global maxima and minima for the function  $H(x)$ . Justify your answers.

5. [8 points] Each part of this problem has four statements, (i)-(iv). For each part, circle all statements which are always true and draw a line through all other statements. Any ambiguous markings will receive no credit.

a. [4 points] Let  $q(t) = A \cos(Bt) + C \sin(Bt)$ , with  $A$ ,  $B$ , and  $C$  constants.

(i)  $q''(t) = -B^2q(t)$ .

(ii) The function  $q(t)$  is concave down everywhere.

(iii) The value of  $q' \left( \frac{\pi}{2B} \right)$  is  $AB$ .

(iv) If  $q'(0) = \pi$  and  $C = 2$ , then  $q(t) = q(t + 4)$  for all values of  $t$ .

b. [4 points] Let  $f(x)$  be a function defined on the closed interval  $[0, 4]$ , such that  $f''(x) > 0$  on the entire interval, and  $f'(x)$  is zero only at  $x = 3$ .

(i)  $f(1) > f(4)$ .

(ii)  $f'(1) < f'(3)$ .

(iii) The point  $(3, f(3))$  is a local maximum.

(iv) Either one or both of  $f(4)$  and  $f(0)$  are a global maximum.

6. [13 points] Let  $f(v)$  be the gas consumption (in liters/km) of a car going at velocity  $v$  (in km/hr). In other words,  $f(v)$  tells you how many liters of gas the car uses to go one kilometer, if it is going at velocity  $v$ . You are told that

$$f(90) = 0.08 \text{ and } f'(90) = 0.0008.$$

- a. [5 points] Let  $g(v)$  be the distance the same car goes on one liter of gas at velocity  $v$ . What is the relationship between  $f(v)$  and  $g(v)$ ? Find  $g(90)$  and  $g'(90)$ .

- b. [5 points] Let  $h(v)$  be the gas consumption in liters per hour. In other words,  $h(v)$  tells you how many liters of gas the car uses in one hour if the car is going at velocity  $v$ . What is the relationship between  $h(v)$  and  $f(v)$ ? Find  $h(90)$  and  $h'(90)$ .

- c. [3 points] How would you explain the practical meaning of  $g'(90)$  to a driver who knows no calculus?

7. [16 points] Let  $f(x) = \ln(x)$ . Use the table of values below for  $g(x)$  and  $g'(x)$  to answer the following questions.

$x$	2	3	4
$g(x)$	1	4	6
$g'(x)$	5	3	2

- a. [4 points] If  $F(x) = f(g(x))$ , find  $F'(4)$ .

- b. [4 points] If  $G(x) = g^{-1}(x)$ , find  $G'(4)$ .

- c. [4 points] If  $H(x) = \tan(g(x))$ , find  $H'(3)$ .

- d. [4 points] If  $E(x) = e^{f(x)g(x)}$ , find  $E'(2)$ .

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8. [12 points] The equation  $(x^2 + y^2)^2 = 4x^2y$  describes a two-petaled rose curve.
- a. [2 points] Verify that the point  $(x, y) = (1, 1)$  is on the curve.
- b. [7 points] Calculate  $dy/dx$  at  $(x, y) = (1, 1)$ .
- c. [3 points] Find the equation of the tangent line to the rose curve at the point  $(x, y) = (1, 1)$ .



9. [12 points] Suppose  $w(x)$  is an everywhere differentiable function which satisfies the following conditions:

- $w'(0) = 0$ .
- $w'(x) > 0$  for  $x > 0$ .
- $w'(x) < 0$  for  $x < 0$ .

Let  $f(t) = t^2 + bt + c$  where  $b$  and  $c$  are positive constants with  $b^2 > 4c$ . Define  $L(t) = w(f(t))$ .

a. [2 points] Compute  $L'(t)$ . Your answer may involve  $w$  and/or  $w'$  and constants  $b$  and  $c$ .

b. [4 points] Using your answer from (a), find the critical points of  $L(t)$  in terms of the constants  $b$  and  $c$ .

c. [6 points] Classify each critical point you found in (b). Be sure to fully justify your answer.