CLASS DISCUSSION: 15 NOVEMBER 2017

FTC

U. Mich. Probs

1.

(12 points) Problems (a) and (b) below are independent of each other.

(a) (6 pts.) Suppose the function r gives the number of customers per day going to a new ice-cream store that just opened near campus. (Assume t is measured in days since the opening and that we are modeling the situation by a continuous function, r.) *IMPORTANT: The answers to (i) and (ii) should include clear units, and should be given using words understandable to someone who has never taken calculus.*

(i) What does $\int_0^{20} r(t)\,dt$ represent?

(ii) If each customer spends on average of \$3.50 in the store, what does the following expression represent?

$$\frac{3.5}{20}\int_0^{20} r(t)\,dt$$

(b) (6 points) If the average value of the function $d(x) = 7/x^2$ on the interval $[1, c]$ is equal to 1, what is the value of c?

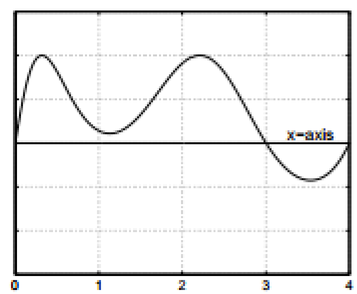
2. (a) What is the average value of the function x^3 on the interval $1 < x < 3$?

(b) If it is known that $\int_1^3 f(x)\,dx = 4$ and $\int_1^3 (f(x))^2\,dx = 5$, then

$$\int_1^3 (1+f(x))^2\,dx = \underline{\hspace{3cm}}$$

(c) A function $f(x)$ has a graph as shown below, and it is known that $\int_0^4 f(x)\,dx = 10$, while the area of the region below the x-axis and above the graph of f is 2. Find

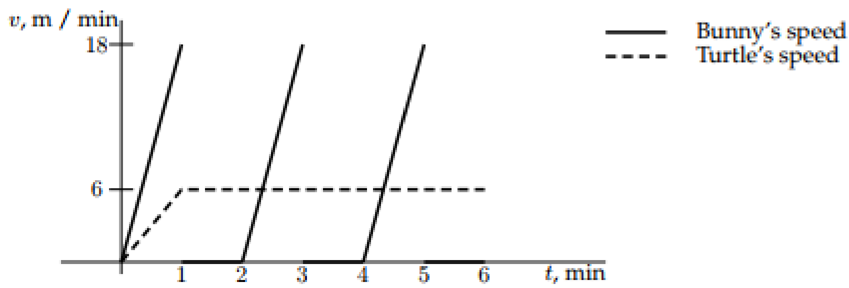
$$\int_0^3 f(x)\,dx = \underline{\hspace{3cm}}$$



(d) The average price (in dollars) of a new house that is A square feet in area is a function $P = f(A)$. What are the units of $dP/dA = f'(A)$.

3.

The Awkward Turtle is competing in a race! Unfortunately his archnemesis, the Playful Bunny, is also in the running. The two employ very different approaches: the Awkward Turtle takes the first minute to accelerate to a slow and steady pace which he maintains through the remainder of the race, while the Playful Bunny spends the first minute accelerating to faster and faster speeds until she's exhausted and has to stop and rest for a minute - and then she repeats this process until the race is over. The graph below shows their speeds (in meters per minute), t minutes into the race. (Assume that the pattern shown continues for the duration of the race.)



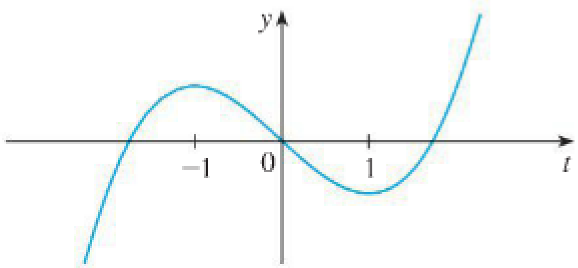
(a) (6 points) What is the Awkward Turtle's average speed over the first two minutes of the race? What is the Playful Bunny's?

(b) (3 points) The Playful Bunny immediately gets ahead of the Awkward Turtle at the start of the race. How many minutes into the race does the Awkward Turtle catch up to the Playful Bunny for the first time? Justify your answer.

(c) (5 points) If the race is 60 meters total, who wins? Justify your answer.

Stewart exercises:

66. Let $F(x) = \int_1^x f(t)\,dt$, where f is the function whose graph is shown. Where is F concave downward?



67. Let $F(x) = \int_2^x e^{t^2}\,dt$. Find an equation of the tangent line to the curve $y = F(x)$ at the point with x-coordinate 2.

68. If $f(x) = \int_0^{\sin x} \sqrt{1 + t^2}\,dt$ and $g(y) = \int_3^y f(x)\,dx$, find $g''(\pi/6)$.

69. If $f(1) = 12$, f' is continuous, and $\int_1^4 f'(x)\,dx = 17$, what is the value of $f(4)$?

70. The **error function**

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\,dt$$

is used in probability, statistics, and engineering.

(a) Show that $\int_a^b e^{-t^2}\,dt = \frac{1}{2}\sqrt{\pi}\,[\operatorname{erf}(b) - \operatorname{erf}(a)]$.

(b) Show that the function $y = e^{x^2}\operatorname{erf}(x)$ satisfies the differential equation $y' = 2xy + 2/\sqrt{\pi}$.

71. The Fresnel function S was defined in Example 3 and graphed in Figures 7 and 8.

(a) At what values of x does this function have local maximum values?

(b) On what intervals is the function concave upward?

(c) Use a graph to solve the following equation correct to two decimal places:

$$\int_0^x \sin(\pi t^2/2)\,dt = 0.2$$

72. The **sine integral function**

$$\mathrm{Si}(x) = \int_0^x \frac{\sin t}{t}\,dt$$

is important in electrical engineering. [The integrand $f(t) = (\sin t)/t$ is not defined when $t = 0$, but we know that its limit is 1 when $t \to 0$. So we define $f(0) = 1$ and this makes f a continuous function everywhere.]

(a) Draw the graph of Si.

(b) At what values of x does this function have local maximum values?

(c) Find the coordinates of the first inflection point to the right of the origin.

(d) Does this function have horizontal asymptotes?

(e) Solve the following equation correct to one decimal place:

$$\int_0^x \frac{\sin t}{t}\,dt = 1$$

73–74 Let $g(x) = \int_0^x f(t)\,dt$, where f is the function whose graph is shown.

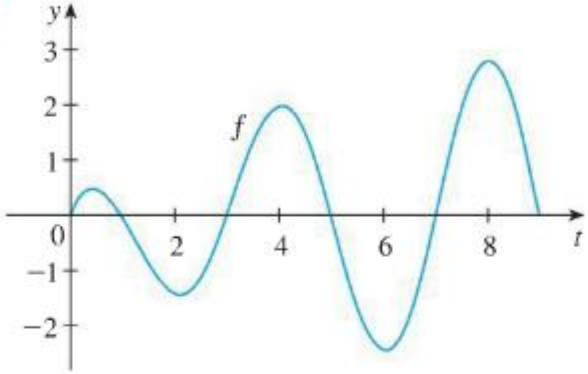
(a) At what values of x do the local maximum and minimum values of g occur?

(b) Where does g attain its absolute maximum value?

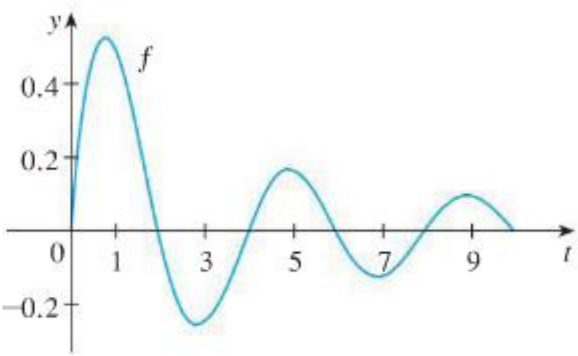
(c) On what intervals is g concave downward?

(d) Sketch the graph of g.

73.



74.



75–76 Evaluate the limit by first recognizing the sum as a Riemann sum for a function defined on [0, 1].

75. $\displaystyle \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{i^4}{n^5} + \frac{i}{n^2} \right)$

76. $\displaystyle \lim_{n\to\infty} \frac{1}{n} \left(\sqrt{\frac{1}{n}} + \sqrt{\frac{2}{n}} + \sqrt{\frac{3}{n}} + \cdots + \sqrt{\frac{n}{n}} \right)$

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77. Justify (3) for the case $h < 0$.

78. If f is continuous and g and h are differentiable functions, find a formula for

$$\frac{d}{dx} \int_{g(x)}^{h(x)} f(t)\, dt$$

79. (a) Show that $1 \leqslant \sqrt{1 + x^3} \leqslant 1 + x^3$ for $x \geqslant 0$.

(b) Show that $1 \leqslant \int_0^1 \sqrt{1 + x^3}\, dx \leqslant 1.25$.

80. (a) Show that $\cos(x^2) \geqslant \cos x$ for $0 \leqslant x \leqslant 1$.

(b) Deduce that $\int_0^{\pi/6} \cos(x^2)\, dx \geqslant \frac{1}{2}$.

81. Show that

$$0 \leqslant \int_5^{10} \frac{x^2}{x^4 + x^2 + 1}\, dx \leqslant 0.1$$

by comparing the integrand to a simpler function.

11–40 Evaluate the integral, if it exists.

11. $\int_1^2 (8x^3 + 3x^2)\,dx$

12. $\int_0^T (x^4 - 8x + 7)\,dx$

13. $\int_0^1 (1 - x^9)\,dx$

14. $\int_0^1 (1 - x)^9\,dx$

15. $\int_1^9 \frac{\sqrt{u} - 2u^2}{u}\,du$

16. $\int_0^1 (\sqrt[4]{u} + 1)^2\,du$

17. $\int_0^1 y(y^2 + 1)^5\,dy$

18. $\int_0^2 y^2\sqrt{1 + y^3}\,dy$

19. $\int_1^5 \frac{dt}{(t - 4)^2}$

20. $\int_0^1 \sin(3\pi t)\,dt$

21. $\int_0^1 v^2 \cos(v^3)\,dv$

22. $\int_{-1}^1 \frac{\sin x}{1 + x^2}\,dx$

23. $\int_{-\pi/4}^{\pi/4} \frac{t^4 \tan t}{2 + \cos t}\,dt$

24. $\int_0^1 \frac{e^x}{1 + e^{2x}}\,dx$

25. $\int \left(\frac{1 - x}{x}\right)^2 dx$

26. $\int_1^{10} \frac{x}{x^2 - 4}\,dx$