

1. (review) Compute $\lim_{x \rightarrow 0} \frac{\sin(3 \cos x)}{\cos(\sin x)}$. Show your work.
2. The parabola $y = x^2 + x + c$ is tangent to the line $y = 3x$. Find c . (Include a picture in your explanation.)
3. Find a parabola, $y = ax^2 + bx + c$, that passes through the point $(1, 4)$ and whose tangent lines at $x = -1$ and $x = 5$ have slopes 6 and -2 respectively.
4. Find equations of any (and all) tangent lines to the parabola $y = x^2 + 1$ that have x -intercept of $-4/3$. Sketch.
5. Archy lives on the x -axis. His position at time t (hours) is

$$s(t) = 4t^3 - 15t^2 + 12t + 1 \text{ (cm)}.$$

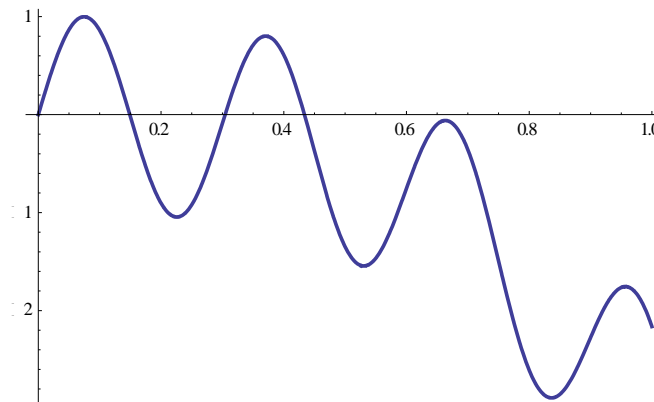
6. Assume that Archie was born at time $t = 0$.
 - (a) What is Archy's *position* at time $t = 1$?
 - (b) What is Archy's instantaneous *velocity* at time $t = 1$?
 - (c) When is Archy moving *toward the left*? (Give one or more time intervals.)
7. (review) The quantity, Q mg, of nicotine in the body t minutes after a cigarette is smoked is given by $Q = g(t)$.
 - (a) Using a complete sentence, interpret the statement $g(20) = 0.36$.
 - (b) Using a complete sentence, interpret the statement $g'(20) = -0.002$.
 - (c) Using the information that you obtained above, estimate $g(23)$.

8. Consider the function $f(x) = 2x^3 - 3x^2 - 12x + 2017$.

Find any and all points (only their x -coordinates) at which the tangent line to $y = f(x)$ is horizontal.

9. Given the following graph of $y = f(x)$, use "geometric differentiation" to sketch the graph of dy/dx .

(If you are curious, the equation of this curve is $y = x^5 + \sin(21x) - 4x^3$)



10. Given $f(x) = x^3 - 6x^2 + 9x - 5$.
 - (a) Find the slope of the tangent line to the curve at $x = -2$. (You may assume that $df/dx = 3x^2 - 12x + 9$.)
 - (b) What is the equation of this tangent line?
 - (c) What is the equation of the normal line at $x = -2$.
 - (d) Find all points where the curve has a horizontal tangent.

11. Does the curve $y = x^3 + x + 1$ ever have a horizontal tangent line? If so, where?

12. Let $G(x) = 2^{bx}$, where b is a non-zero constant.

Albertine, our friend, informs us that $d/dx (2^{bx}) = b 2^{bx} \ln 2$.

Find $G^{(2017)}(x)$. Show your reasoning! An answer with no justification will earn little or no credit.

13. Let $f(x) = x|x|$ be defined on the real line. Is f differentiable at $x = 0$? True or False? Explain.

14. Which of the following graphs (a) – (d) could represent the slope at every point of the function graphed in figure 2.6?

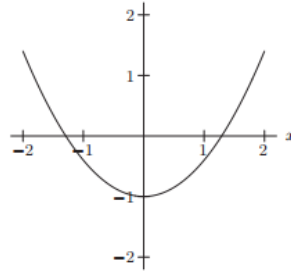
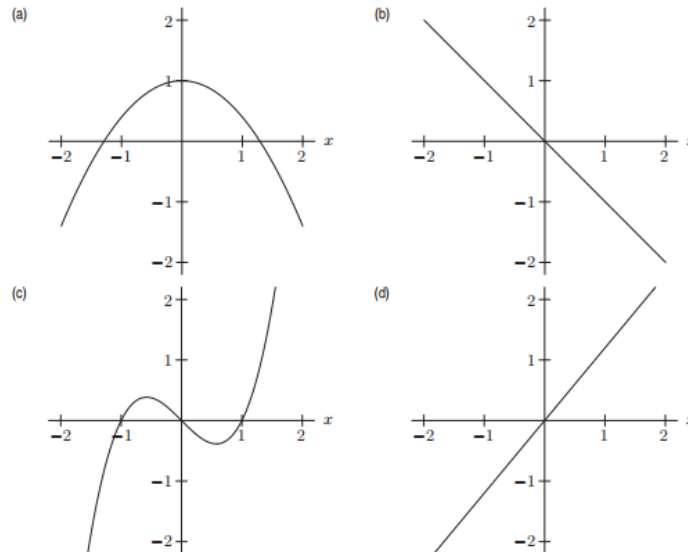
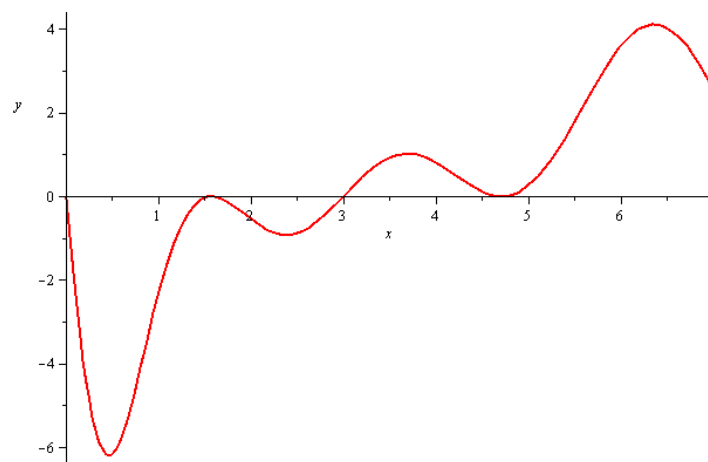


Figure 2.6



15. Using the process of “geometric differentiation,” sketch the graph of the derivative of the function $y = G(x)$ whose graph is given below:



16.

Which of the following graphs (a)–(d) could represent the slope at every point of the function graphed in Figure 2.8?

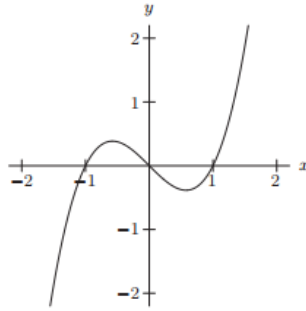
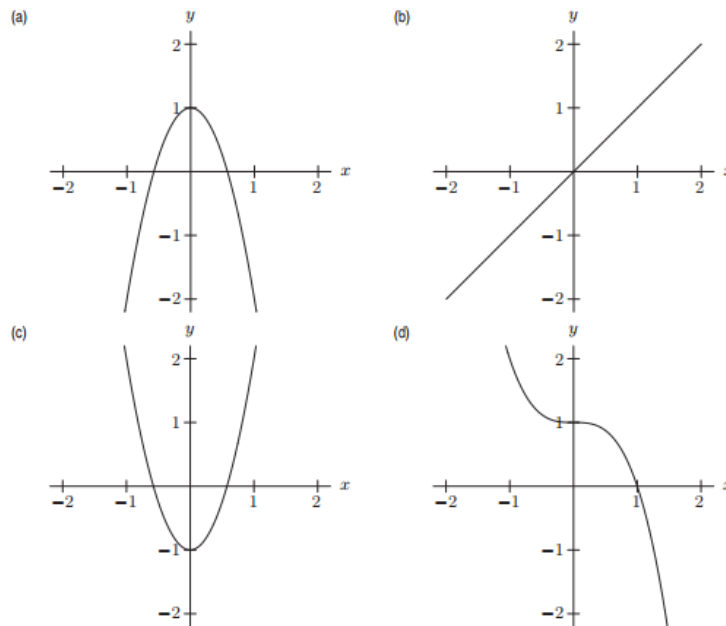


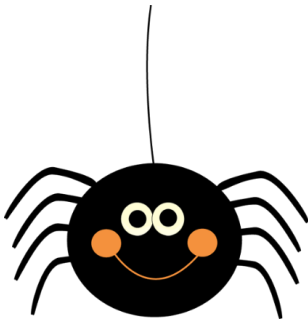
Figure 2.8



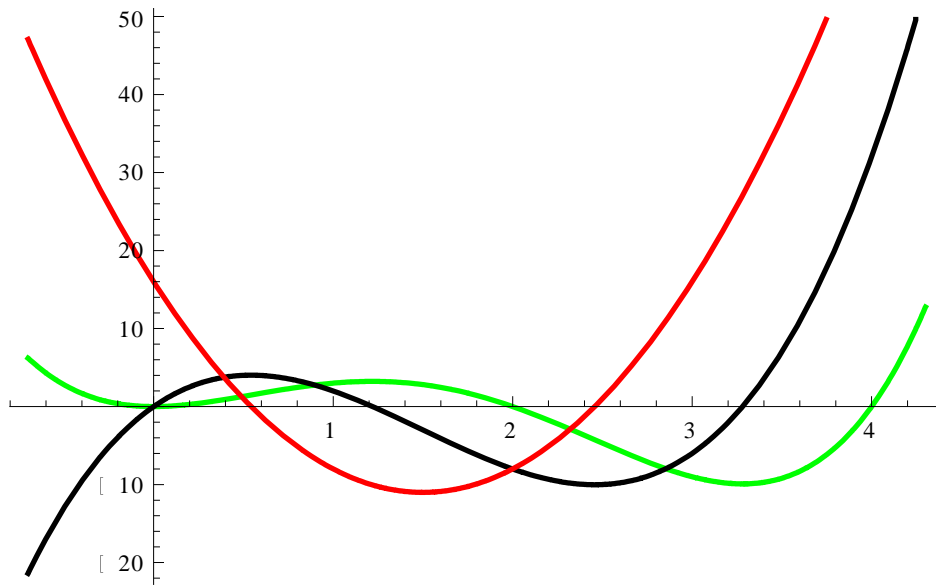
17. Assume that Charlotte, who chooses to live on the y-axis, is located at

$$y(t) = 3 \cos t + 4 \sin t \text{ cm at time } t \text{ (measured in minutes).}$$

- Find her *position* at times $t = 0$, $t = \pi/2$, and $t = \pi$.
- Find her *velocity* when $t = 0$, $t = \pi/2$, and $t = \pi$ minutes.
- Find her *acceleration* when $t = 0$, $t = \pi/2$, and $t = \pi$.



18. Archy lives on the x -axis. Graphs of his *position*, *velocity* and *acceleration* during the time interval $-0.7 < t < 4.3$ appear below. Which is which? Explain.



19. Let $f(x) = x^4 - ax^2$.
- Find all possible critical points of f in terms of a .
 - If $a < 0$, how many critical points does f have?
 - If $a > 0$, find the x and y coordinates of all critical points of f .
20. Given $f(x) = x^6 - 3x^5$ on the interval $[-1, 4]$.
- Find all critical points of f .
 - Determine on which intervals f is increasing.
 - Find and classify all local and global extrema of f .
 - Sketch the graph of f using the above information.
21. Given $f(x) = g(x) = \frac{x}{x+1}$.
- What is the domain of g ?
 - Find all critical points of g .
 - Determine on which intervals g is increasing.
 - Find and classify all local and global extrema of g .
 - Sketch the graph of f using the above information.

Which of the following graphs (a)–(d) could represent the slope at every point of the function graphed in Figure 2.11?

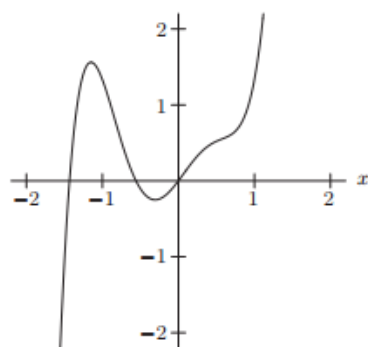
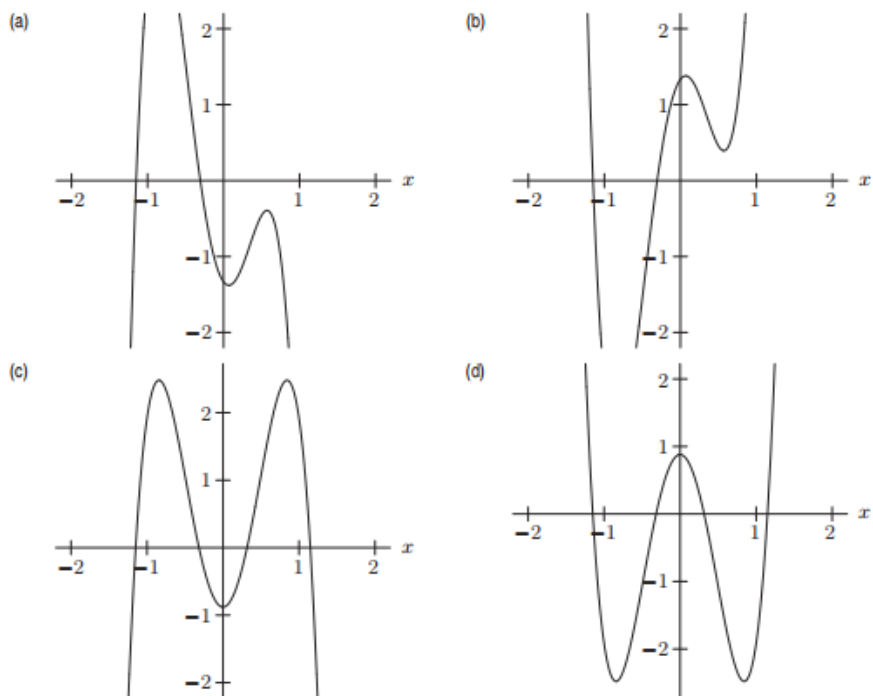


Figure 2.11



23.

[10 points] Elphaba the squirrel has been involved in some questionable activity of late and hence is being very cautious. She has made eye contact with a human standing near her multiple times and is getting anxious that the human is observing her. Let $f(x)$ be Elphaba's anxiety (in "anxious units") after making eye contact with the human for a total of x seconds. Elphaba will panic and run when her anxiety reaches 100 anxious units.

From across the room, the human, Erin, is in fact observing Elphaba while pretending to read a newspaper. The total amount of time Elphaba has spent making eye contact with Erin is a function of the number of times that Erin looks up from the newspaper. Let $g(n)$ be the total amount of time, in seconds, that Erin and Elphaba have spent making eye contact if Erin has looked up from her newspaper n times.

a. [2 points] Using a complete sentence, give a practical interpretation of the expression $f^{-1}(3) = 10$. Be sure to include units.

b. [3 points] Below is the first part of a sentence that will give a practical interpretation of the equation

$$f'(25) = 2.$$

Complete the sentence so that the practical interpretation can be understood by someone who knows no calculus. Be sure to include units in your answer.

If Elphaba has already made eye contact with Erin for a total of 25 seconds and she makes eye contact for an additional 0.3 seconds, then

c. [2 points] Given that $(f^{-1})'(99) = 7$ and $f(62) = 99$, approximate the total length of time Elphaba has to spend making eye contact with Erin before she will panic and run.

d. [3 points] Which of the following sentences gives a correct interpretation of the quantity $g^{-1}(f^{-1}(50))$? *Circle the ONE best answer.*

- i. When Erin has looked up from her newspaper 50 times, Elphaba's anxiety is at $g^{-1}(f^{-1}(50))$ anxious units.
- ii. When Erin has looked up from her newspaper 50 times, Erin and Elphaba have spent $g^{-1}(f^{-1}(50))$ seconds making eye contact.
- iii. If Erin has looked up from her newspaper $g^{-1}(f^{-1}(50))$ times then Elphaba's anxiety is 50 anxious units.
- iv. If Erin and Elphaba have made eye contact for a total of 50 seconds then Erin has looked up from her newspaper $g^{-1}(f^{-1}(50))$ times.
- v. When Erin and Elphaba have made eye contact for a total of 50 seconds then

When I was four years old they tried to test my IQ. They showed me a picture of three oranges and a pear. They asked me, "Which one is different and does not belong?" They taught me different was wrong.

- Ani DiFranco