CLASS DISCUSSION: 30 AUGUST 2017

(CALCULATOR FREE)

1. *(algebra review)* For which non-zero value of *k* will the following quadratic equation have *only one* real root?

$$7x^2 + kx + 3k = 0$$

2. Consider the following rational functions. For each function, determine the limiting behavior as $x \to \infty$. Briefly explain how you arrived at your answers.

(a) $y = \dfrac{(2x-5)^2}{x^2}$

Answer: As $x \to \infty$, $y \to$ _____

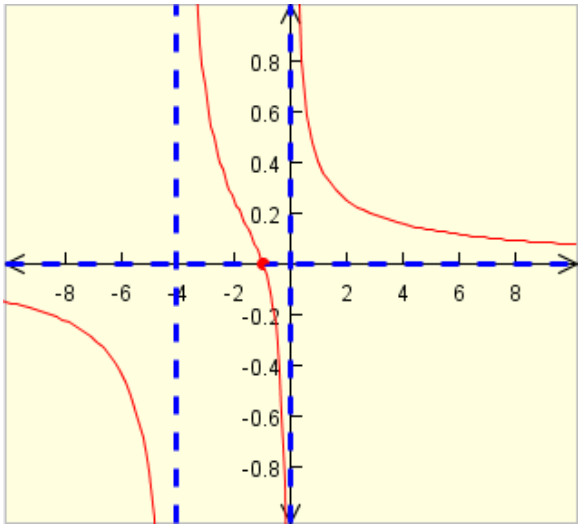
(b) $y = \dfrac{(x+5)^2(x-11)^3}{(x-9)(x+13)(x-1)(7x-44)}$

Answer: As $x \to \infty$, $y \to$ _____

(c) $y = \dfrac{99(x-15)(3x+11)}{(x-9)^2(19x+13)}$

Answer: As $x \to \infty$, $y \to$ _____

3. Find an *equation* of a rational function whose graph is given below:



4. Sketch the graph of each of the following rational functions (that includes all the significant properties):

(*a*) $f(x) = (x-1)^2(x-3)^3(3x+5)$

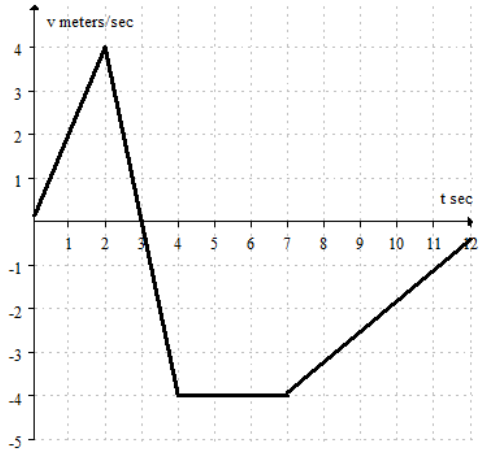
$(b)\ f(x)=\dfrac{x^2+5}{(x+1)^2}$

$(c)\ f(x)=\dfrac{(x-1)(x+2)}{x-3}$

$(d)\ f(x)=\dfrac{(x-3)^2(x-4)^3(x-5)^5}{x^2(2x^2+x+1)^4}$

5. (*University of Michigan* problem)

The graph below shows the velocity of a bug traveling along a straight line on the classroom floor.



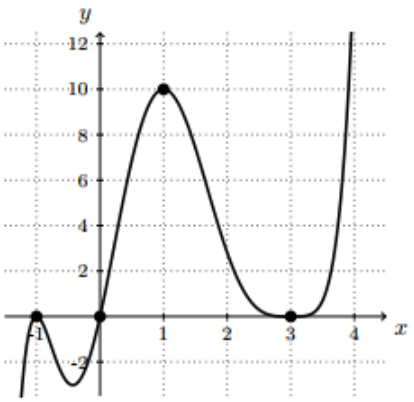
At what time(s) does the bug turn around?

A) At 3 seconds.
B) At 2 seconds and again at 7 seconds.
C) At 4 seconds and again at 7 seconds.
D) Never.

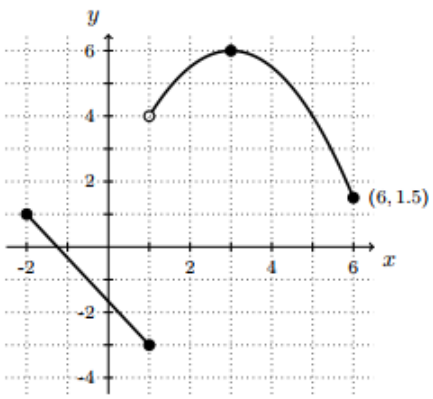
6. (*University of Michigan* problem)

[11 points] Consider the graphs of $y = k(x)$ and $y = \ell(x)$ given below:

GRAPH OF $y = k(x)$



GRAPH OF $y = \ell(x)$



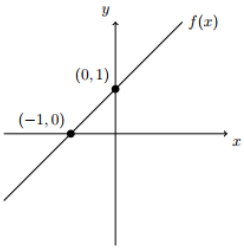
You must **show your work** in both parts of this problem to receive full credit. Write your final answers *in the spaces provided.*

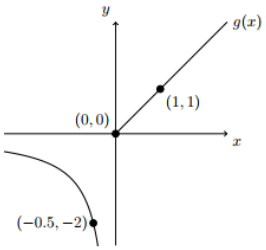
a. [5 points] Find a formula for $k(x)$, assuming $k(x)$ is a polynomial of degree seven with zeros at $x = -1$, $x = 0$ and $x = 3$.

b. [6 points] Find a piecewise-defined formula for $\ell(x)$ on $[-2, 6]$, given that the graph of $y = \ell(x)$ is made up of a line and a parabola.

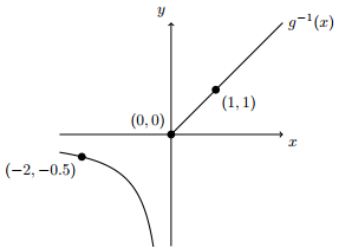
7. (*University of Michigan* problem

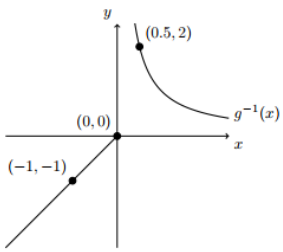
[5 points] A portion of the graphs of $y = f(x)$ and $y = g(x)$ are given below. **You do not need to show any work for this problem.**

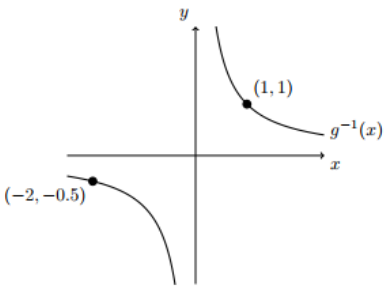


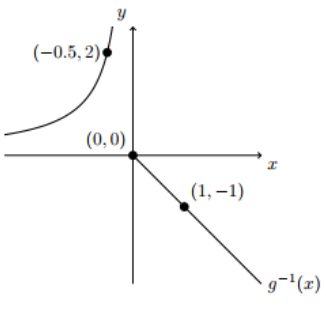


a. [2 points] Assume that $g(x)$ is an invertible function. Which of the following could be the graph of $y = g^{-1}(x)$? **Circle exactly one of the four graphs below.**

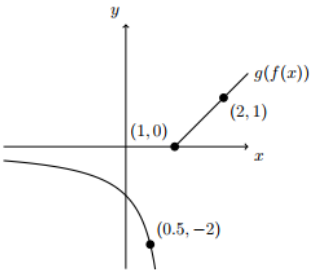


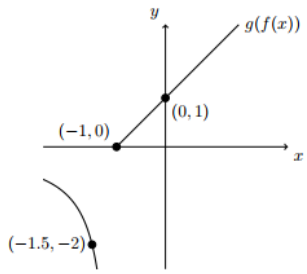


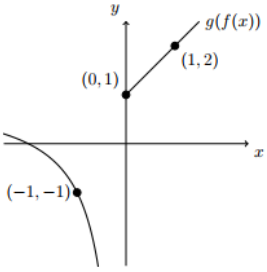


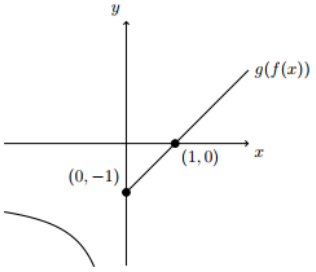


b. [3 points] Which of the following could be the graph of $y = g(f(x))$? **Circle exactly one of the four graphs below.**









8. Compute each of the following limits. Explain your reasoning. Do not use calculators.

(a) $\lim_{x\to\infty} \dfrac{(x+11)^2(3x-7)^3}{(2x^2+4)^4(x+2017)}$

(b) $\lim_{x\to\infty} \dfrac{1+\sqrt{x}}{5+x^2}$

9. (*University of Michigan* problem)

[15 points] The number of hemlock trees in the southern Appalachian mountains is declining as a result of an infestation of hemlock woolly adelgids (a kind of insect).

- There are $H(d)$ *healthy* hemlock trees in the southern Appalachian mountains d days after January 1, 2013.
- There are $I(d)$ *infested* hemlock trees in the southern Appalachian mountains d days after January 1, 2013.

Note that all hemlock trees are considered healthy unless they are infested. Be sure to write your final answers *in the spaces provided.*

a. [2 points] Let $J(w)$ be the number of *healthy* hemlock trees in the southern Appalachian mountains w *weeks* after January 1, 2013. Find a formula for $J(w)$ in terms of any or all of the functions H and I.

$J(w) =$ _____

b. [3 points] Let $F(d)$ be the fraction of the hemlock trees in the southern Appalachian mountains that are *infested* d days after January 1, 2013. Find a formula for $F(d)$ in terms of any or all of the functions H and I.

$F(d) =$ _____

c. [4 points] Let $K(d)$ be the total number of hemlock trees in the southern Appalachian mountains, in *thousands*, d days after January 1, 2013. Find a formula for $K(d)$ in terms of any or all of the functions H and I.

$K(d) =$ _____

d. [3 points] The number of hemlock trees I that are *infested* in the southern Appalachian mountains is *inversely proportional* to the cube of the total amount of money M (in millions of dollars) that the government spends combating the spread of the adelgids. Write a formula for I in terms of M, assuming that there were 2,000 infested trees when the government had spent 3 million dollars. You must **show your work** for this part.

$I =$ _____

e. [3 points] The number of hemlock woolly adelgids A (in millions) is also a function of the amount of money M (in millions of dollars) that the government spends to try to preserve the hemlock trees, and is given by:

$$A(M) = \frac{4}{M}$$

for $M \geq 4$. Find the equation of the horizontal asymptote of $y = A(M)$, **and** interpret this horizontal asymptote in practical terms.

The equation of the horizontal asymptote is _____

10. Consider the rational function *F* defined by

$$F(x)=\frac{15x^3+x^2-6x}{6x^2+x-2} \text{ if } x\neq 1/2 \text{ and } x\neq -2/3$$

(a) Find the lim F(x) as x → ∞ if it exists. Explain.

(b) Find the lim F(x) as x → -∞ exist? Explain.

11. Let y = g(x) be defined as follows

$$g(x)=\begin{cases} 3-x \text{ when } x<2 \\ 2 \text{ if } x=2 \\ \frac{x}{2} \text{ if } x>2 \end{cases}$$

Sketch the curve.

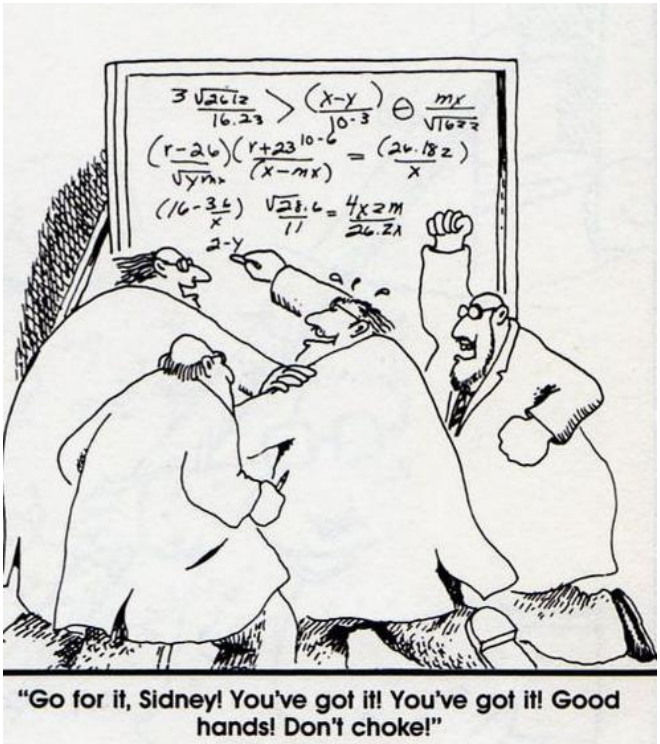
12. Does the limit of g(x) as x→∞ exist?

$$g(x)=\frac{3x^2-4x+1}{x^4-1}$$

If so, find it; if not explain!

To be pleased with one's limits is a wretched state.

- **Johann Wolfgang von Goethe** (1749 - 1832)



"Go for it, Sidney! You've got it! You've got it! Good hands! Don't choke!"