MATH 161 CLASS DISCUSSION 13 SEPT 2017

1. The cost of extracting T tons of ore from a copper mine is $C = F(T)$ dollars. Using a complete sentence that avoids mathematical terminology, explain the meaning of $F(2000) = 300{,}000$. (Include appropriate units.)

2. Albertine travels from Chartres to Mt. Saint Michelle at an average speed of 50 km/hr. She returns to Chartres at an average speed of 60 km/hr. What is Albertine's *average speed* during the roundtrip?

3. The expression

$$\frac{V(3) - V(1)}{3 - 1}$$

represents

(a) The average rate of change of the radius with respect to the volume when the radius changes from 1 inch to 3 inches.
(b) The average rate of change of the radius with respect to the volume when the volume changes from 1 cubic inch to 3 cubic inches.
(c) The average rate of change of the volume with respect to the radius when the radius changes from 1 inch to 3 inches.
(d) The average rate of change of the volume with respect to the radius when the volume changes from 1 cubic inch to 3 cubic inches.

4. A paperback book (definitely not a valuable calculus textbook, of course) is dropped from the top of Dennison hall (which is 40 m high) towards a very large, upward pointing fan. The average velocity of the book between time $t = 0$ and later times is shown in the table of data below (in which t is in seconds and the velocities are in m/s).

between $t = 0$ seconds and $t =$	1	2	3	4	5
average velocity is	-5	-10	-11.67	-9	-7.2

a. [8 points] Fill in the following table of values for the height $h(t)$ of the book (measured in meters). Show how you obtain your values.

t	0	1	2	3	4	5
$h(t)$	40	______	______	______	______	______

Which of the following represents the rate at which the volume is changing when the radius is 1 inch?

(a) $\dfrac{V(1.01) - V(1)}{0.01} = 12.69 \text{ in}^3$
(b) $\dfrac{V(0.99) - V(1)}{-0.01} = 12.44 \text{ in}^3$
(c) $\displaystyle\lim_{h\to 0}\left(\frac{V(1+h) - V(1)}{h}\right) \text{ in}^3$
(d) All of the above

5.

Which of the following expressions represents the slope of a line drawn between the two points marked in Figure 2.5?

(a) $\dfrac{F(\Delta x) - F(x)}{\Delta x}$ (b) $\dfrac{F(x+\Delta x) - F(x)}{\Delta x}$ (c) $\dfrac{F(x+\Delta x) - F(x)}{x}$

(d) $\dfrac{F(x+\Delta x) - F(x)}{x + x - \Delta x}$ (e) $\dfrac{F(x+\Delta x) - F(x)}{x + \Delta x}$

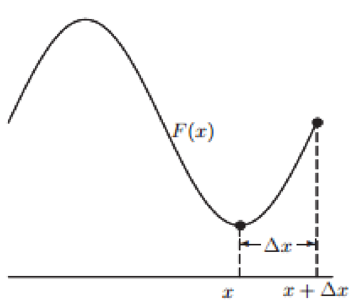


Figure 2.5

6.

[12 points] Suppose that when you merge onto the highway the blue car in front of you is moving at 55 mph. Immediately after you merge, the driver of the blue car speeds up until, after five minutes, it is going 85 mph. Then, during the next five minutes it slows down to 55 mph again. This process then repeats over the following 10 minutes, with the blue car speeding up to 85 mph and then decreasing to 55 mph again.

a. [6 points] Assuming the speed of the blue car follows a sinusoidal pattern, on the axes below draw a well-labeled sketch of two periods of a function $v(t)$ which outputs the speed of the car t minutes after you merge onto the highway.

7. *(University of Michigan problem)*

A runner competed in a half marathon in Anaheim, a distance of 13.1 miles. She ran the first 7 miles at a steady pace in 48 minutes, the second 3 miles at a steady pace in 28 minutes and the last 3.1 miles at a steady pace in 18 minutes.

a) Sketch a well-labeled graph of her distance completed with respect to time.

b) Sketch a well-labeled graph of her velocity with respect to time.

8.

[15 points] During the winter, the town of Waterville uses salt to keep the roads from freezing. Let $S = f(T)$ be the amount of salt, in tons, used on the roads of Waterville on a day when the average temperature is T °F. Let $C = g(S)$ be the cost, in thousands of dollars, of S tons of salt. Assume that both f and g are invertible functions that are differentiable everywhere.

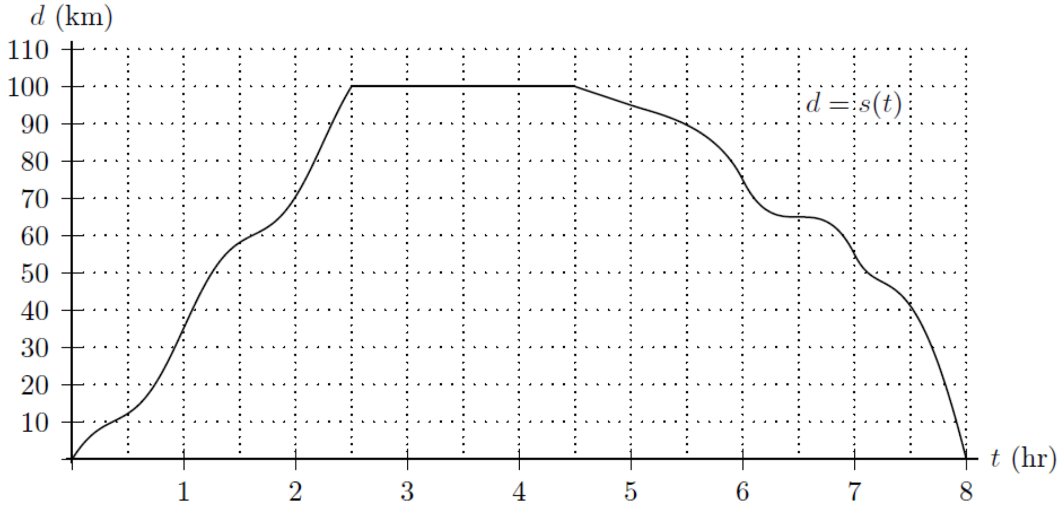
a. [3 points] Interpret the equation $f^{-1}(4) = 9$ in the context of this problem. Use a complete sentence and include units.

b. [3 points] Interpret the equation $g(f(7)) = 2$ in the context of this problem. Use a complete sentence and include units.

c. [2 points] Yesterday, the average temperature in Waterville was w °F. Give a single mathematical expression equal to the average temperature, in °F, on a day when Waterville uses twice as much salt on the roads as it did yesterday.

9.

[8 points] A ship's captain is making a round trip voyage between two ports. The ship sets sail from Port Jackson at noon, arrives at Port Kembla some time later, waits there for a while, and then returns to Port Jackson. Let $s(t)$ be the ship's distance, in kilometers, from its starting point of Port Jackson, t hours after noon. A graph of $d = s(t)$ is shown below.



Remember to include units where appropriate.

a. [1 point] How far is Port Kembla from Port Jackson?

Answer: ____________________

b. [1 point] How long does the ship wait in Port Kembla?

Answer: ____________________

c. [1 point] Sometime after 5 PM, there is a time when the ship's instantaneous velocity is 0 km/hr. At what time does this occur?

Answer: ____________________

d. [2 points] What is the ship's average speed during the return trip from Port Kembla to Port Jackson?

I turn away with fright and horror from the lamentable evil of functions which do not have derivatives.
– Charles Hermite (in a letter to Thomas Jan Stieltjes)

Charles Hermite (1822 – 1901)