# MATH 161 PRACTICE FINAL EXAM B

**PART I** (*3 pts each*) ***Answer any ten of the following eleven questions.*** You need not justify your answer. You may answer all eleven to obtain extra credit.

1. $\lim\limits_{n\to\infty} \dfrac{(2n^3+n+1)^2}{(2n^2+5)^3} =$

2. $\displaystyle\int_{-5}^{5} \sqrt{25-x^2}\, dx =$

3. $\dfrac{d}{dx} \ln(\cos x) =$ .

4. Let $h(x) = \displaystyle\int_0^x \sin(e^{t^2})\, dt$ . Compute h′(1).

5. $\dfrac{d^{55}}{dx^{55}} \sin x =$

6. Assume that *a*, *b*, and *c* are non-zero constants. Then

$$\int_0^1 \frac{(at+b)^2}{c}\, dt =$$

7. Find an *anti-derivative* of

$$\frac{\cos\left(\frac{1}{x}\right)}{x^2}$$

8. Find an anti-derivative of:

$$\frac{x}{x+5}$$

9. Let $G(x) = e^{9x} + 13$. Find a general formula for $G^{(k)}(0)$, the $k^{th}$ derivative of $G$ evaluated at x = 0, where $k \geq 1$.

10. Suppose that *f* is defined and twice differentiable on the interval (0, 98). If f ' (9) = 0 and f "(9) = 43, what, if anything, can you conclude about the point x = 9? (For example, is it a point of inflection, a local maximum, a global minimum, etc?)

11. $\dfrac{d}{dx}\ x^{\ln x}$

**PART II** *(7 pts each)*

*Answer any 9 of the following 11 problems. You may answer more than eight for extra credit.*

1. Use implicit differentiation to find dy/dx , where *y* is given implicitly as a function of *x* by the following equation:

$$x^3 + 4xy - 3y^{\frac{4}{3}} = 2x$$

2. Show that the tangents to the curve $y = \pi \dfrac{\sin x}{x}$ at the points x = π and x = -π, respectively, are perpendicular.

3. The impedance Z (ohms) in a series circuit is related to the resistance R (ohms) and reactance X (ohms) by the equation $Z = \sqrt{R^2 + X^2}$. If R is increasing at 3 ohms/sec and X is decreasing at 2 ohms/sec, at what rate is Z changing when R = 10 ohms and X = 20 ohms?

4. Given the function f (x) = x ln(2x) – x on the closed interval $\left[\dfrac{1}{2e}, \dfrac{e}{2}\right]$, find the global extrema, and points of inflection and use this information to sketch the graph.

5. For which value or values of the constant *k* will the curve $y = x^3 + kx^2 + 3x - 4$ have exactly one horizontal tangent?

6. Using an appropriate substitution, evaluate the following definite integral:

$$\int_1^4 \frac{dx}{2\sqrt{x}(1+\sqrt{x})^2}$$

7. Sketch the graph of the function $y = \frac{(x+1)^2}{1+x^2}$. Locate any and all local extrema and points of inflection.

8. You are designing a 1000 $cm^3$ cylindrical can whose manufacture will take waste into account. There is no waste in cutting the aluminum for the side, but the top and bottom of radius *r* will be cut from squares that measure 2r units on a side. The total amount of aluminum used up by the can will therefore be

$$A = 8r^2 + 2\pi rh.$$

Find the ratio of *h* to *r* for the most economical can.

9. It costs Albertine *c* dollars to manufacture and distribute backpacks. If the backpacks sell at *x* dollars each, the number sold is given by

$$n = \frac{a}{x-c} + b(100-x)$$

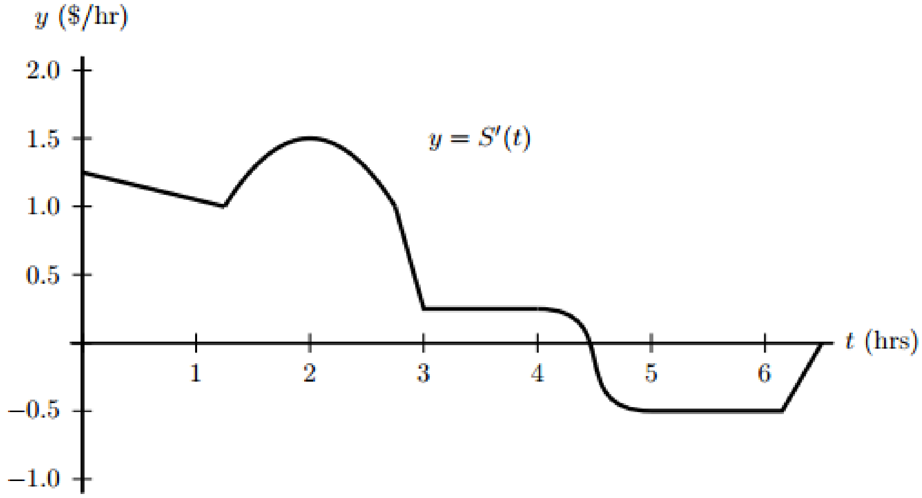
where *a* and *b* are positive constants.

What selling price will yield a maximum profit for Albertine?

10. Using the FTC, find the area bounded by the curve $y = x^2(x-4)^2$ and the x-axis. Sketch.

*11. (University of Michigan)*

[8 points] Suppose that a new company named Calculus Knowledge, which provides calculus consulting work, was posted on the New York Stock Exchange over the summer. Let $S(t)$ be a continuous and differentiable function that models the price, in dollars, of one share of Calculus Knowledge stock $t$ hours after 9:30 am on October 6, 2014.
The graph of $S'(t)$ for $0 \leq t \leq 6.5$ is shown below.




Note: The graph above is the graph of $S'(t)$. It is **not** the graph of $S(t)$.

**a.** [2 points] Estimate when the price of the stock is rising most quickly on October 6, 2014.

**Answer:** ____________________

**b.** [2 points] According to the model $S(t)$, at which of the times 10 am, 11 am, 12 noon, and 1 pm was the price of one share of Calculus Knowledge stock the lowest on October 6, 2014?
*Circle* ONE *time or circle* CANNOT BE DETERMINED *if the answer cannot be determined from the information provided.*

10 am   11 am   12 noon   1 pm   CANNOT BE DETERMINED

**c.** [2 points] On which, if any, of the following intervals does it appear that the function $S(t)$ is always decreasing? *Circle* ALL *correct choices or circle* NONE OF THESE *if appropriate.*

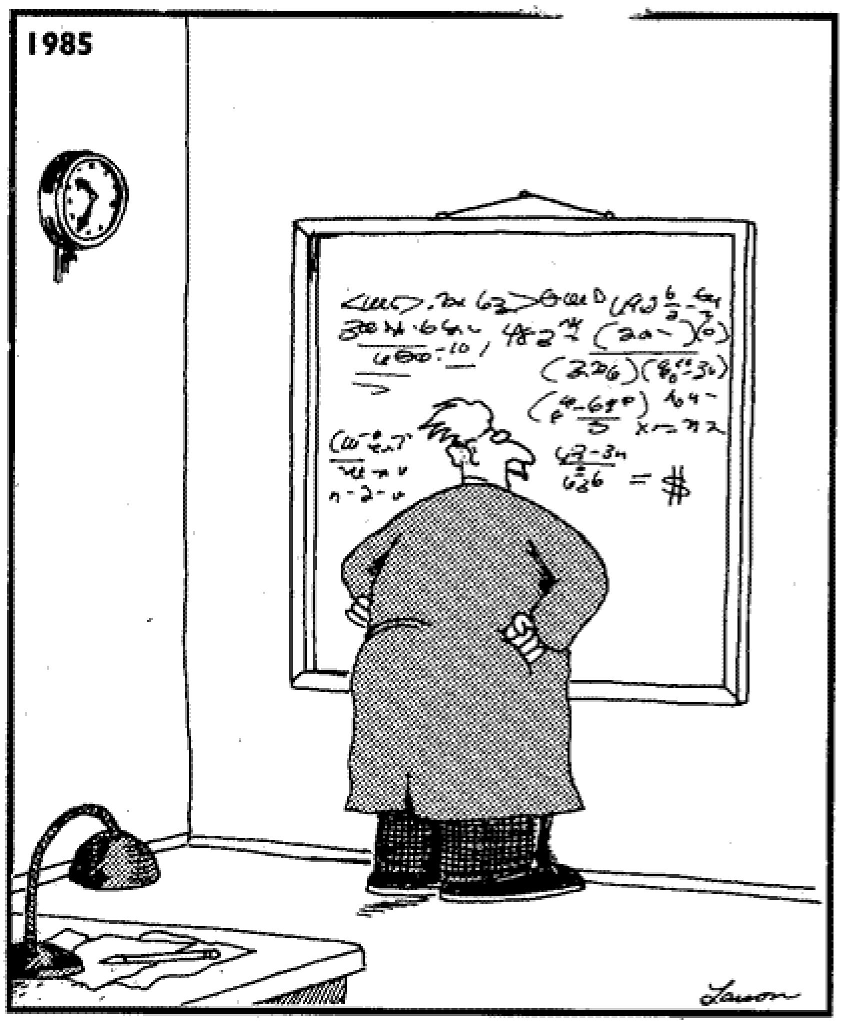
$0 < t < 1$   $2 < t < 3$   $4 < t < 5$   $5 < t < 6$   NONE OF THESE

**d.** [2 points] On which, if any, of the following intervals does it appear that $S(t)$ is linear? *Circle* ALL *correct choices or circle* NONE OF THESE *if appropriate.*

$0 < t < 1$   $1 < t < 2$   $3 < t < 4$   $5 < t < 6$   NONE OF THESE

*In the beginning (if there was such a thing), God created Newton's laws of motion together with the necessary masses and forces. This is all; everything beyond this follows from the development of appropriate mathematics methods by means of deduction.*

- Albert Einstein




Einstein discovers that time is actually money.