# MATH 161 Solutions: Quiz x 1 December 2017

1. Albertine is studying Newton’s method. She is trying to find the roots of a cubic polynomial

p(x) = x3 – x + 1 = 0

1. *[3 pts]* Albertine has determined that there must be a root between x = -2 and x = -1. How can she be so certain?

*Solution: First observe that p(x) is continuous, since it is a polynomial.*

*Then note that p(0) = 1 > 0 and p(-2) = -5 < 0.*

*Hence, by the Intermediate Value Theorem, p(x) must have a root in the interval (0, -2).*

1. *[7 pts]* Albertine’s initial guess is x0 = -1. Using Newton’s method, find x1 and x2. Express each answer to 3 significant digits.

*Solution: Letting* x0 = -1*, we find that* y0 = f(x0) = f(-1) = 1.

*Also*

*So the equation of the tangent line to y = p(x) at is*

*Letting y = 0, we find:*

*Repeating this process to find x2 ;*

*So the equation of the tangent line to y = p(x) at is*

*.*

*Letting y = 0, we find:*

1. *[Stewart problem, 10 pts]* Find

*Solution: Let .*

*The limit as is an indertminate form:*

*To use L'Hôpital’s rule, we convert this to an indeterminate form of type*

*Now, is of the form*

*Invoking L'Hôpital’s rule,*

*Now, since this new limit is of the form , we may invoke L'Hôpital’s rule once again, viz.*

*=*

*Now since .*

1. *[Stewart problem, 10 pts]* Evaluate the indefinite integral

*Solution: Let u = 1 + ex. Then du = ex dx. Thus*

1. *[Stewart problem, 10 pts]* Evaluate

*Solution: Let w =*

*Also, as x varies from Thus*

*Extra Credit [Stewart problem, 10 pts*:

*Solution:*

*Let t =*

*But just as much as it is easy to find the differential*

*[derivative] of a given quantity, so it is difficult to find the*

*integral of a given differential. Moreover, sometimes we cannot*

*say with certainty whether the integral of a given quantity can be*

*found or not.*

– Johann Bernoulli